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Fractal descriptors in the Fourier domain applied to color texture analysis

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The present work proposes the development of a novel method to provide descriptors for colored texture images. The method consists of two steps. First, we apply a linear transform in the color space of the image aiming at highlighting spatial structuring relations among the color of pixels. Second, we apply a multiscale approach to the calculus of fractal dimension based on Fourier transform. From this multiscale operation, we extract the descriptors that are used to discriminate the texture represented in digital images. The accuracy of the method is verified in the classification of two color texture datasets, by comparing the performance of the proposed technique to other classical and state-of-the-art methods for color texture analysis. The results showed an advantage of almost 3% of the proposed technique over the second best approach.

Fractal objects constitute a particular category of non-linear dynamic system. Essentially, fractals are geometric objects which do not obey the classical Euclidian rules. Besides, they are characterized by the self-similarity, that is, we observe some geometrical and/or statistical patterns which repeat themselves along different observation scales. In a true fractal, the self-similarity is observed at infinite observation scales. The inherent composition rules of fractals yield to some interesting characteristics. For instance, fractals present non-proportionality between cause and effect. Still they present infinite complexity (complexity in the sense of observed details) and they show a high dependence level from initial conditions. Such characteristics allow for fractals are also to be considered as a chaotic system. This physical interpretation collaborated for authors, such as Mandelbrot,¹ to suggest the use of fractals to model objects found in the nature. More recently, Manoel et al.² developed a method for using fractal theory in the extraction of features from natural objects represented in a digital image. This is an important problem in computational vision. Bruno et al.³ applied the technique to a shape recognition problem. Here, we propose the development and study of a novel fractal-based method for the analysis of color texture. We verify the efficiency of the proposed approach in the solution of a color texture classification problem.

I. INTRODUCTION

Texture is a visual attribute which is present in most of the nature images. The texture quantification, identification, and classification are the most important problems that are defined in computer vision as the study of pixel patterns in an image region. Although this attribute is naturally processed by natural vision and easily comprehended by humans, there is no formal definition for it. Indeed, textures are complex visual patterns formed by arrangements of pixels, regions, or even set of patterns formed by other visual attributes, such as shape or color. These patterns can be composed by completely distinct factors, such as pixel organization or even its disorganization. In fact, depending on the context, even the noise can be considered as a sort of texture. These characteristics of the texture attribute make it special and hard to be well defined.

Along the last years, several methods have been developed for the analysis of textures. Such interest in texture analysis methods may be comprehended by the richness of the texture attribute in images, which was analyzed in pattern recognition problems.

Basically, the texture analysis methods can be divided into 4 categories,⁴ that is, the structural methods, in which the texture is described as a set of primitives well defined; statistical methods, in which the texture is represented through non-deterministic measures of distribution; spectral methods, based on the analysis in the frequency domain; and model-based methods, based on the mathematical and physical modelling of the texture image.

Among the model-based methods, the fractal model has presented a large projection recently in the description of textures in a wide number of problems.⁵–⁷ Most of these methods employ the fractal dimension direct or indirectly for the representation of the texture. In recent years, however, a family of methods³,⁸–¹⁰ was developed through extracting a set of features from the fractal modelling, unlike the fractal dimension which is only a unique number. Generically speaking, these methods provide the called fractal descriptors, capable of representing a texture with a higher degree of richness than the simple fractal dimension.

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The ability of fractal features in the description of textures is related to the nature of fractality concept. The fractal dimension measures the complexity of a structure, which in turn corresponds to important physical properties of a material, such as the roughness, the reflectance, etc. Finally, such properties are strong stimuli in our visual identification of texture images by allowing the classification of objects based on their texture aspects. In this way, fractal theory becomes a worthy tool in the automation of this process.

Actually, the use of term “fractal” in such kind of application may bring some controversy, if the analyzed images are not real fractals. Because, fractals are only mathematical entities lacking any perfect representation in the real world. The interested reader may appreciate this discussion for example in a letter exchange involving Avnir et al. and Mandelbrot. Inasmuch as this debate is not in any way finished, the most acceptable approach in the literature is that presented in Ref. 13. There, objects from real world are measured through fractal metrics even when they get away from fractal concept. In this case, the fractal measures act as a complexity metric of the real world object.

This work proposes a novel technique for the extraction of fractal descriptors based on the Fourier fractal dimension method from colored textures. The color is an important attribute in texture images, mainly those extracted from natural scenes. The method consists in representing the color image in a new space color through the linear transform described in Ref. 16. This transform aims at emphasizing the relation between the pixels color and their spatial distribution. Posteriorly, we apply the method for the calculus of fractal dimension by the Fourier transform. In this method, the dimension is obtained from a curve relating the power spectrum of the Fourier transform and the frequency. Instead of simply using the dimension value, this work proposes the use of the whole curve to provide the descriptors of the texture.

The proposed method takes some important advantages over classical fractal signature techniques like that based on wavelets or multifractal. One of such advantages is that the technique here presented gathers information from frequency domain inherently, allowing the capturing of details and patterns which escapes from the conventional spatial analysis. Besides, the space used allows the expression of colors and spatial distribution of pixels as being a related entity. This relation is important in many applications involving color texture. Moreover, and not less relevant, the method shows a simple computational implementation and presents a low computational effort.

The performance of the proposed technique is tested in comparison with other classical and state-of-the-art methods for color texture analysis, namely, chromaticity moments, histogram ratio, and multispectral Gabor. The comparison is achieved in the classification of two color texture datasets.

This work is divided into 8 sections. Section II addresses the definitions and theoretical aspects of fractal theory. Section III describes the Fourier fractal dimension. Section IV presents the concept of fractal descriptors. Section V describes the proposed method. Section VI shows the experiments employed. Section VII discusses the results and the last one does the conclusions.
where \( H^s(F) \) is the \( s \)-dimensional measure of \( F \), defined through

\[
H^s(F) = \lim_{\epsilon \to 0} H^s_\epsilon(F),
\]

where

\[
H^s_\epsilon(F) = \inf \left\{ \sum_{i=1}^{\infty} |U_i|^{\frac{s}{n}} : U_i \text{ is an } \epsilon\text{-cover of } F \right\}.
\]

**III. FOURIER FRAC TAL DIMENSION**

The literature still shows alternative definitions for the fractal dimension. Among these definitions, one of the most important is the Fourier fractal dimension. For the calculus of a measure \( \mu \) in \( \mathbb{R}^n \), the Fourier transform is defined by

\[
\mathcal{F}(\mu(u)) = \int_{\mathbb{R}^n} e^{iux}d\mu(x),
\]

where \( u \) is a generic subset of \( \mathbb{R}^n \) and \( x \) is the Fourier space counterpart of \( u \).

In the following, we employ an analogy from the classical mechanics for the definition of the fractal dimension. We use the concept of \( s \)-potential of a mass distribution \( \mu \) over a point \( x \) in \( \mathbb{R}^n \), given by

\[
p_v(x) = \int \frac{1}{|x-y|}d\mu(y),
\]

where \( y \) is an auxiliary variable.

By still extending the Physics analogy, the potential energy \( e_s \) may be obtained through

\[
e_s(\mu) = (2\pi)^n c \int \mathcal{F}(p_v)(u)\mathcal{F}(\mu(u))du,
\]

in which \( c \) is a constant dependent on \( s \) and \( n \) and \( \bar{x} \) is the complex conjugate of \( x \). In this way,

\[
e_s(\mu) = (2\pi)^n c \int |u|^{n-s} |\mathcal{F}(\mu(u))|^2du.
\]

From a theorem developed in Ref. 19, if there is a mass distribution \( \mu(u) \) on the set \( S \subset \mathbb{R}^n \) for which the expression (10) is finite for some value(s) of \( s \), the Hausdorff dimension of \( S \) has its lower limit in \( s \). Particularly, if \( |\mathcal{F}(\mu(u))| \leq b|u|^{-t/2} \), for a constant value \( b \), then \( e_s(\mu) \) always converges if \( s < t \). The greatest \( t \) for which there is a mass distribution \( \mu \) on \( S \) is called the Fourier fractal dimension of \( S \).

For practical purposes, the method for the calculus of the Fourier fractal dimension described in Ref. 14 is applied to a gray-scale (intensity) image \( I \).

In this case, we have a 2D real-valued image \( I(i, j) \) with size \( N \times N \) and the Fourier transform \( \tilde{I} \) is expressed through

\[
\tilde{I}(u, v) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} I(i, j) \exp(-2\pi i (ui + vj)),
\]

where \( u \) and \( v \) are, respectively, the horizontal and the vertical frequencies. The total frequency \( f \) is given by \( f = \sqrt{u^2 + v^2} \).

Russ demonstrates that there is an exponential relation between the frequency \( f \) and the power spectrum \( P \) in the Fourier spectrum of \( I \)

\[
P \propto f^{-\alpha}.
\]

As for the exponentiation parameter \( \alpha \) may be used for the estimation of the fractal dimension \( D \) of the texture in \( I \). In the practice, \( \alpha \) is calculated as being the slope of the curve \( \log(P) \times \log(f) \). The dimension is easily estimated through

\[
D = \frac{\alpha + 6}{2}.
\]

In digital image applications, the Fourier transform is calculated by classical optimized techniques, such as fast Fourier transform. The Fourier spectrum is divided into radial rings (corresponding to frequency bands). Thus, the variable \( P \) in the previous expression corresponds to the power spectrum averaged over each ring and the frequency \( f \) corresponds to the average distance of each ring from the center of the spectrum. Figure 1 illustrates the process. Figure 2 illustrates the dimension calculated by this practical method.

**IV. FRACTAL DESCRIPTORS**

Although the fractal dimension is a good descriptor for textures, shapes, contours, etc., they become inefficient in tasks which require a greater precision in the description of the object. Fractal descriptors have arisen with the aim of filling this gap and provide a more precise technique for the characterization of the image. Figure 3 shows graphically the importance of fractal descriptors.

The first known work applying the concept of fractal descriptor is Manoel et al. which uses the technique named multiscale fractal dimension (MFD) to obtain the descriptors. In this approach, the fractal dimension from the object...
is inferred at different observation scales. In Manoel et al., the authors used the Minkowski sausage method for the calculus of the fractal dimension. In this method, the object is dilated by a variable radius and the area of the dilated object (number of pixels) is called the dilation area . The fractal dimension is estimated from the slope of the curve \( \log(A(r)) \) against \( \log(r) \). Instead of simply obtaining the dimension, the authors used the whole \( \log(A(r)) \) curve to compose the descriptors by the measures extracted from the curve, such as the peaks, the area under the curve, etc.

In turn, in Refs. 3 and 9, we have an application of MFD in which the derivative of \( \log(A(r)) \) is used in order to provide the fractal descriptors. Bruno et al. still applies the MFD to the analysis of textures, mapped onto surfaces and using the volumetric Minkowski sausage method.

V. PROPOSED METHOD

This work proposes a novel method for the extraction of fractal descriptors from colored texture images.

Several methods have been described in the literature for the extraction of features from colored textures with the aim of solving problems such as classification and segmentation in different application fields. However, many of such works do not take into account the spatiality of the color, that is, the relation between the color of a pixel and its position in the image or in a specific neighborhood.

In order to deal with this situation, Geusebroek et al. proposed an interesting method based on physical characteristics of colors. Roughly speaking, the method consists in a linear transform from the original color space into another physical space. In the special case in which the original space is RGB (red-green-blue) like it is in our case, the transformation is represented by the simple expression,

\[
\begin{bmatrix}
\tilde{E}_k \\
\tilde{E}_{kk} \\
\tilde{E}_{kkk}
\end{bmatrix} =
\begin{bmatrix}
0.06 & 0.63 & 0.31 \\
0.19 & 0.18 & -0.37 \\
0.22 & -0.44 & 0.06
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix},
\]

where \( R, G, \) and \( B \) are the original color channels and \( \tilde{E}_k, \tilde{E}_{kk}, \) and \( \tilde{E}_{kkk} \) are the transformed channels. \( \tilde{E}_k \) corresponds to the convolution of color energy (wavelength) with a Gaussian, \( \tilde{E}_{kk} \) represents the same convolution with the first derivative of a Gaussian, while \( \tilde{E}_{kkk} \) is the same with the second derivative of Gaussian filter. In Ref. 24 the authors apply the classical Gabor filters to the transformed color space obtaining interesting results.

In this work, initially, we apply the transform described to the texture image. In the following, for each one of the channels \( \tilde{E}_k, \tilde{E}_{kk}, \) and \( \tilde{E}_{kkk} \) we extract fractal descriptors based on the Fourier fractal dimension described in Sec. III. For the calculus of these descriptors, instead of simply calculating the fractal dimension by Eq. (13), we use all the values of \( \log(P) \) in the curve, that is, the logarithm of the whole power spectrum. Thus, we apply a multiscale transform to

FIG. 2. (Color online) Calculus of the fractal dimension of textures. (a) Original texture. (b) Power spectrum. (c) Log-log curve of power spectrum \( \times \) frequency.

FIG. 3. (Color online) Illustration of the richness of texture fractal descriptors. At left, two textures with similar fractal dimensions. At right, the fractal descriptors for each texture and the clear visual distinction between them.
the curve, with the aim of capturing the fractal behavior at different observation scales, in a similar manner to that described in Ref. 9.

Essentially, a multiscale transform is a mapping from the original signal \( u(t) \) onto a function \( U(b,a) \), where \( b \) is related to the original variable \( t \) and \( a \) is the scale parameter. The literature presents several approaches for the calculus of multiscale transform.\(^{25,26}\) Based on empirical results, we opted for the use of space-scale approach. In this solution, the transform is formally represented through

\[
\{ (b,a)|a \in \mathbb{R}, a > 0, b \in \{ U^1(t,a) \}_{zc} \}, 
\]

in which \( zc \) represents the zero-crossings of \( u \), and \( U^1(t,a) \) expresses the convolution of \( u(t) \) with the first derivative of the Gaussian \( g^1_{a^*} \), given by

\[
U^1(t,a) = u(t) \ast g^1_{a^*}(t),
\]

where \( a \) denotes the smoothing parameter of Gaussian, referenced in the most of textbooks as \( \sigma \). The best results were achieved by calculating the derivative through the Fourier property and projecting \( U(b,a) \) onto the axis corresponding to \( a = 0 \). This multiscale representation based on Eqs. (15) and (16) uses the Gaussian kernel concept. It states, using a result from partial derivative theory, that the Gaussian filter
associated with the first derivative is the unique operation capable of representing an image or signal under different scales without adding any spurious element in the process. More details may be seen in Ref. 25.

Finally, we concatenate the fractal descriptors from each channel, generating the final color Fourier fractal descriptor. The steps of the method are depicted in Figure 4, while Figure 5 summarizes the algorithm process.

By joining the power of fractal theory and more specifically fractal descriptors in the description of natural textures and the efficiency of the color approach described in Ref. 24, we obtain a powerful descriptor for natural colored textures. Such descriptor is capable of capturing complex patterns in the texture, which are capital for a complete and precise identification of a real scene. Figure 6 shows in a simple example the potentiality of texture discrimination which is present in the proposed descriptor may be seen even visually.

VI. EXPERIMENTS

The performance of the proposed technique is tested by the classification of samples from VisTex,27 a classical dataset of colored textures and USPTex, a dataset developed in the research group of the authors, which is composed by

```
1 Load image
2 Extract E_lambda channels using the transform matrix
3 For each channel
4   Calculate Fourier transform
5   Divide spectrum into radial rings (frequency band)
6   For each ring
7       power spectrum = squared magnitude
8   end
9 Fractal descriptors = multiscale(log(power spectrum)xlog(frequency))
10 end
11 Final descriptors = concatenate(fractal descriptors)
```

FIG. 5. Proposed method generic algorithm.

FIG. 6. (Color online) The ability of the proposed descriptors in the discrimination of texture classes. Left, we see texture images from two classes and their respective descriptors. Right, the descriptors are plotted in a same graph, showing visually the high discrimination potential.
images of natural textures, photographed in high resolution. Figures 7 and 8 show some image samples from each dataset.

The dimensionality of proposed descriptors is proportional to the dimension of the image. Thus, for the VisTex dataset, the signatures are composed by 120 descriptors, while in USPTex we use 132 descriptors. We have not used other resolutions for descriptors, once such approach may compromise the fractality measure, in which all multiscale levels have equal importance for the description of the texture.

The results obtained were compared with the use of other classical methods for descriptors of colored textures, that is, color Gabor, color histogram ratio, and chromaticity moments. The descriptors are classified by the well known K-Nearest Neighbor (KNN) method, with \( k = 1 \) (empirically determined) and using a 10-fold cross-validation process. The comparison is done in terms of the correctness rate with its confidence interval and the confusion matrices for each descriptor and dataset.

VII. RESULTS

In a first moment, we tested the classification performance of some metrics extracted from MFD curve, as it is done in the original MFD work. Here, we employed some fractal and statistical metrics, e.g., fractal dimension of channels 1, 2, and 3, mean, standard deviation, kurtosis, skewness, second and third order moments, and the combination of these measures. Table I shows the correctness rate and associated error for each approach in Vistex dataset. Thus, we see that the use of whole MFD curve in the composition of descriptors provided the best result. From now, we show results for the use of the whole curve in the tested datasets.

Initially, we see in Table II the global correctness rate for each compared descriptor for Vistex and USPTex dataset. It is clearly noticeable that the proposed Fourier method presented the best result in the classification of both datasets. The proposed technique presented an advantage of 2.8% in Vistex dataset and 2.7% in USPTex over Gabor method, the second best technique in this experiment.

In Table III, we present the confusion matrices for each descriptor method. In this matrix, each raw (or column) represents a class and the value in row \( i \) and column \( j \) expresses the number of objects of class \( i \), but classified as being from class \( j \). The ideal method (with a 100% correctness rate) must present a diagonal confusion matrix.

For the Vistex dataset, the numbers correspond to the following classes (exemplified in Figure 7): 1-Bark,
2-Fabric, 3-Food, 4-Metal, 5-Sand, 6-Tile, and 7-Water. As expected, the greater the correctness rate, more the confusion matrix presents diagonal aspect. Particularly, the proposed descriptor presented its best performance in classes 3,

![Image samples from the USP-Tex dataset.](image)

**TABLE I.** Percentage correctness rate and respective confidence interval in the classification of Vistex dataset using statistical metrics extracted from MFD curve.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Correctness rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>FD Channel 1</td>
<td>30.51 ± 0.20</td>
</tr>
<tr>
<td>FD Channel 2</td>
<td>33.11 ± 0.23</td>
</tr>
<tr>
<td>FD Channel 3</td>
<td>35.38 ± 0.14</td>
</tr>
<tr>
<td>Mean</td>
<td>32.46 ± 0.21</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>26.62 ± 0.17</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>20.12 ± 0.27</td>
</tr>
<tr>
<td>Skewness</td>
<td>29.54 ± 0.39</td>
</tr>
<tr>
<td>2th Moment</td>
<td>26.62 ± 0.30</td>
</tr>
<tr>
<td>3th Moment</td>
<td>29.87 ± 0.40</td>
</tr>
<tr>
<td>Combined</td>
<td>93.83 ± 0.21</td>
</tr>
<tr>
<td>Whole Curve</td>
<td>95.12 ± 0.10</td>
</tr>
</tbody>
</table>

2-Fabric, 3-Food, 4-Metal, 5-Sand, 6-Tile, and 7-Water. As expected, the greater the correctness rate, more the confusion matrix presents diagonal aspect. Particularly, the proposed descriptor presented its best performance in classes 3,

**TABLE II.** Percentage correctness rate and respective confidence interval in the classification of the tested datasets by the compared descriptors.

<table>
<thead>
<tr>
<th>Method</th>
<th>Vistex</th>
<th>USPTex</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment</td>
<td>68.83 ± 0.33</td>
<td>32.06 ± 0.05</td>
</tr>
<tr>
<td>Histogram</td>
<td>78.89 ± 0.21</td>
<td>41.49 ± 0.18</td>
</tr>
<tr>
<td>Gabor</td>
<td>92.53 ± 0.18</td>
<td>86.47 ± 0.04</td>
</tr>
<tr>
<td>Fourier</td>
<td>95.12 ± 0.10</td>
<td>88.83 ± 0.07</td>
</tr>
</tbody>
</table>

2-Fabric, 3-Food, 4-Metal, 5-Sand, 6-Tile, and 7-Water. As expected, the greater the correctness rate, more the confusion matrix presents diagonal aspect. Particularly, the proposed descriptor presented its best performance in classes 3,
4, and 5. Good results are also observed in classes 1 and 6. Specially, in class 6, the proposed technique presented a relevant advantage over Gabor method. Gabor misclassified elements from class 6 as being from classes 1, 2, and 5. This is explained by the self-similarity present in these classes. The fractal method, as expected, captured more faithfully the self-similar nuances. The confusion matrices from USPTex were represented in a different graphical manner, by using surface figures in Figure 9. In this representation, each position in the matrix is represented by a surface point and the height of the point, according to the legend on the axis, determines the value in that position. Looking at each figure, we observe that the matrix from the Fourier method presented a more continuous diagonal with the highest points, justifying the greater number of samples correctly classified. The Gabor method presented bad results around class 110.

VIII. CONCLUSION

This work proposed a novel technique for the calculus of descriptors for colored textures. The method uses the Fourier spectral dimension associated with the spatial color transform proposed in Ref. 16. Initially, the transform is applied to the original texture images in RGB space. Following, the classical Fourier transform is applied to the image and the values in the curve $\log(\text{powerspectrum}) \times \log(\text{frequency})$ are used as descriptors for the texture.

The accuracy of the proposed method was verified by comparing other classical techniques in color texture analysis. The results demonstrated the power of the novel technique once Fourier descriptors presented the greater precision in the classification of two complex datasets. Specially, the proposed technique showed a great efficiency in capturing self-similar patterns in the texture.

The results suggest strongly that color Fourier fractal descriptors are an interesting alternative to be used in the solution of problems in which the description of an object is primordial, like tasks involving segmentation and classification of objects represented by their texture.

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