Characterization of westward propagating signals in the South Atlantic from altimeter and radiometer records

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ABSTRACT

Radar altimeter data from TOPEX/Poseidon and Jason-1 and microwave radiometer data from TRMM/TMI are used to investigate the large-scale variability between 10.5°S and 35.5°S in the South Atlantic Ocean. The proposed method for the analysis of the longitude–time diagrams of the cross-correlation between SSH and SST anomalies shows that the variability in mid latitudes is a blend of first-mode baroclinic Rossby waves and propagating mesoscale eddy-like structures. The estimated phase speed of the wave (\(c_p\)) and propagation speed of the eddies (\(c_e\)) are similar. In 70% of the cases, the absolute difference between \(c_p\) and \(c_e\) is less than 11%. In 40% of the cases this difference is less than 5%. Statistical results indicate that in the case of eddies, as the thermocline deepens the sea surface temperature rises and vice-versa. However, planetary waves show more complex, yet self-consistent results. In lower latitudes (10.5°S–15.5°S), the shallower thermocline and the weak thermal gradients impose a zero phase lag between temperature and height, similar to eddies. Poleward of those latitudes, sea surface temperature and height are in quadrature of phase. This indicates that geostrophic advection of the relatively stronger thermal gradient is performed by Rossby waves.

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1. Introduction

Apart from the large-scale non-propagating signal, the most energetic signal in the sea surface temperature (SST) and sea surface height (SSH) data is composed of meso- to large-scale anomalies, approximately 300–1000 km, that propagate westward and oscillate with annual to intra-annual frequencies. According to Robinson (1983) there are two processes able to explain variability on these spatial and temporal scales: Rossby waves and eddies. Previous studies (Chelton & Schlax, 1996; Cipollini et al., 1997; Polito & Cornillon, 1997; Polito & Liu, 2003) showed that the largest portion of these propagating signals is associated with first-mode baroclinic Rossby waves. However, recent studies performed by Chelton et al. (2007); Chelton, Schlax, et al. (2011) conclude that most of the observed westward propagating signals previously classified as Rossby waves cannot be explained by existing wave theories and should be reinterpreted as nonlinear mesoscale eddies, i.e. the westward propagating variability is dominated by mesoscale eddies. On the other hand, they do not rule out the role of Rossby waves in this variability completely. The latest results obtained by Chelton, Schlax, et al. (2011) reinforce the discussion about the predominance of eddies on Rossby waves and it is the main motivation for the present study in attempting to separate the contributions of these two classes of phenomena.

The meso- and large-scale oceanic variability obeys the quasi-geostrophic vorticity equation whose linearized solution supports Rossby waves. The formation of eddies is associated with instability processes, thus they depend on the nonlinear terms of the equation. Despite the fact that they are dynamically two distinct phenomena, the main observational problem is how to distinguish between Rossby waves and eddy-like features, because eddies propagate with speeds similar to the phase speed of baroclinic Rossby waves. Therefore, in longitude-time diagrams, eddies and waves would generate somewhat similar patterns of propagation. There are some important differences: eddies are generally isolated features, more intermittent when compared with waves. In addition, they have a significant meridional propagation speed and impose a specific phase lag between SST and SSH. The working hypothesis is that the eddy and wave signals are distinguishable through the analysis of the 2D cross-correlation between SST and SSH. In that, both eddies and waves imprint their statistically relevant length and time scales in their auto-correlation matrices. However, the cross-correlation selects commonalities between the two spectra and retains the phase lag information.

Altimetry and thermal satellite records are, respectively, the observational data most often used to identify wave and eddy signals. The method developed here uses SSH and SST data to quantify the relative contribution of each portion of the signal, wave and eddy, to the total variance. The objective is to characterize the westward propagating signals in terms of their dominant physical processes and propagation speeds obtained from SSH and SST data. Our focus is the simultaneous analysis of first-mode baroclinic Rossby waves and eddies in the two datasets, SSH and SST. To do that, a method based on the analysis of the zonal-temporal cross-correlation matrices was developed.
1.1. Observations of westward propagation

Described in the 1930s by Carl-Gustav Rossby, oceanic Rossby waves were observed for the first time from in situ data by Emery and Magara (1976); McWilliams and Flierl (1976), and studied in numerical models by BARNIER (1986); KIRTMAN (1997); Subrahmanyam et al. (2009), and many others. The advent of precise satellite altimetry in the 1990s provided an observational basis that allows the identification of propagating features through measurements of the sea surface height anomaly (SSHA). Using data from TOPEX/Poseidon (T/P) satellite altimeter Polito and Cornillon (1997); Cipollini et al. (1997); Polito and Liu (2003) observed and quantified the main parameters associated with the propagation of baroclinic Rossby waves in the Atlantic Ocean. The discrepancy between the phase speeds obtained from altimeter observations (Chelton & Schlax, 1996) and the linear theory elicited an extension of the classical theory by Killworth et al. (1997); Killworth and Blundell, 2003a, 2003b. In these extensions the authors suggested that the presence of baroclinic east–west shear flow and a slowly varying of the topography induces variations in the potential vorticity gradient able to modify the phase speed of the waves.

Baroclinic Rossby waves have also been observed in SST records from microwave radiometers. The Along-Track Scanning Radiometer (ATSR) onboard the European Remote Sensing satellites (ERS-1; ERS-2) and the TRMM Microwave Imager (TMI) onboard the Tropical Rainfall Measuring Mission (TRMM) are the radiometers most used to observe westward propagating signals in the ocean (Challenor et al., 2004; Cipollini et al., 1997; Hill et al., 2000). First mode baroclinic Rossby waves generate perturbations in the thermocline/poncton depth on the order of three times greater than at the surface, but with the opposite phase. These vertical displacements generate thermocline-induced signals that can reach the surface by two distinct but non-exclusive reasons. One is that the vertical motion induced by the passage of the wave can advect colder waters closer to the surface. Turbulent mixing acting on a thinner upper layer could expose these colder temperatures to the surface. The other is that, because the dominant \( \beta \)-induced propagation of long Rossby waves is mostly zonal (Polito & Cornillon, 1997), the slopes of the surface and thermocline produce a north–south geostrophic flow. This, in the presence of a meridional gradient temperature in the background, results in periodic SST variations caused by horizontal advection. In that, the phase lag between SST and SSH in the two cases would be different and thus the dominant process can, in principle, be identified. Killworth et al. (2004) used the same idea of horizontal and vertical mechanisms to discuss the observation of planetary waves in satellite-derived chlorophyll-a concentrations.

Early satellite-based studies used radiometers to estimate eddy characteristics from local maxima in the SST gradients with a somewhat circular symmetry (e.g., Auer, 1987; Halliwell & Mooers, 1979; Hooker & Brown, 1994). Altimeters were introduced in the study of eddies after the Geodetic Satellite Exact Repeat Mission (Geosat ERM) whose cross-track resolution was sufficient to resolve the mesoscale vortices (Fu & Zlotnicki, 1989). Estimates of eddy characteristics from altimetry satellites were presented by Stammer (1997); Ducet et al. (2000); Wang et al. (2003); Fu (2006). Crawford et al. (2000) observed mesoscale eddies with propagation speeds similar to that of Rossby waves in the Alaskan Stream from altimeter records. This merged eddy-wave signal is suggested from observations that show westward propagating signals with characteristics that fit neither the classical theory for Rossby waves nor its extensions. Challenor et al. (2001) observed wave-like SSH anomalies in the North Atlantic Ocean that propagate westward with little meridional deviations. Chelton, Schlax; et al. (2011) made a comprehensive analysis of eddies in mid latitudes using a methodology based on the Okubo–Weiss parameter to identify and track vortex cores based on their high relative vorticity and low strain rate. The authors suggest that previous observations may have erroneously identified eddies as waves.

Despite the similarity in phase speed or propagation speed, there are important differences between Rossby waves and eddies. Rossby waves are forced by the wind stress at the eastern boundaries or by its curl in the open ocean, may be dispersive or non-dispersive and transport a large quantity of potential energy westward (Chelton & Schlax, 1996). In principle, they do not transport a significant amount of mass, and have an important role in the maintenance of the western boundary currents (Anderson & Gill, 1975). Pedlosky (2003) defines a wave as “a moving signal, typically moving at a rate distinct from the motion of the medium.” The term “eddy” is generally used to different types of variable flow (Robinson, 1983), here, we use the definition for eddies proposed by Cushman-Roisin (1994): “Eddy is a closed circulation roughly circular and relatively persistent, that is, the turnaround time of a parcel of fluid embedded in the structure is shorter than the time during which the structure remains identifiable.” If a relatively high correlation between SSH and SST is observed in certain oceanic regions, then it is reasonable to assume that advection (horizontal or vertical) is the mechanism that connects the variabilities in both fields. To that effect, two physical processes are foreseen: (1) westward zonal propagation of the waves generates a dominant meridional geostrophic flow. This meridional flow can result in a horizontal advection of the SST anomalies in the presence of a background meridional temperature gradient; and (2) the vertical flow generated by the wave propagation on the interface can modify the SST by vertical advection of the isotherms, to the point where they enter the mixed layer and, via turbulent mixing, imprint a low temperature signal at the surface. The simplest model that supports Rossby waves is the quasi-geostrophic model (see Gill, 1982, Section 12.2), in which we can separate the total velocities in geostrophic and ageostrophic parts. The geostrophic velocities are, in this formulation, dominant and perpendicular to the direction of wave propagation. The physical process (1) refers to the term \( \nu_y \) where \( \nu_y = \frac{\partial w}{\partial y} \). The vertical advection in process (2) refers, in a 1 layer model, to the notion of the thermocline \( w_y \) where, to first order, \( \nu_y = \frac{\partial p}{\partial y} \). If \( \eta \) can be represented by a sinusoidal function and the first hypothesis is true, the SST will be correlated to the SSH with a phase lag of 90° in zonal space, because of the x-derivate. If the second hypothesis is true, a significant correlation between SSH and SST at zero zonal lag is expected. Killworth et al. (2004) showed that phase in observed westward propagating signals depends on the response time of the chlorophyll to the vertical advection as a consequence of exposure of phytoplankton to sunlight and/or upwelling of nutrients from the deep layers. Although the same conceptual model is considered in the following analysis, in the present case this response time should be zero because we are observing temperature and not a biological process.

2. Data processing

The altimeter data used here are the along-track TOPEX/Poseidon and Jason-1 (T/J) sea surface height anomalies distributed by the Physical Oceanography Distributed Active Archive Center for the period January 1998 to December 2007. TOPEX/Poseidon was launched in August 1992 and kept in operation until October 2005. Jason-1, its successor, was launched in December 2001 with the same characteristics of the TOPEX/Poseidon. TOPEX and Jason use a radar operating in the frequency of 13.6 GHz to determine the SSH between the latitudes of 66°N and 66°S with precision and accuracy of \( \pm 2.4 \) and \( \pm 1.4 \) cm, respectively (Benada, 1997). The along-track SSHA data were interpolated to a regular grid of \( 1^\circ \times 1^\circ \times 9.9156 \) days. The interpolation uses a method similar to the one developed by Polito et al. (2000), based on the local 3D (x, y, t) auto-correlation.

Our focus in the present study are phenomena with annual and semiannual periods and 300–1000 km of characteristic length. With a sampling period of less than 10 days, the temporal resolution of
these altimeters is clearly not an issue, these phenomena will be very oversampled in time. The spatial resolution is 7 km along-track with the largest longitudinal gap between tracks of 315 km at the equator dropping to zero at approximately 66°; within our study region they range between 100 and 200 km. Longer wavelengths occur in lower latitudes, where even considering the maximum inter-track gaps, we have enough spatial resolution, regardless of interpolation scheme. The shorter wavelengths occur in the higher latitudes, where the satellite sampling mesh becomes tighter and where again we have a sufficient sampling rate, even using only the highest precision altimeter data.

The SST data were obtained by the TRMM Microwave Imager (TMI) through the Remote Sensing Systems website for the period between January 1998 to December 2007, in a space-time resolution of 0.25° × 0.25° × 1 day. TMI is a passive microwave radiometer launched in November 1997 through a partnership between National Aeronautics and Space Administration (NASA) and Japan Aerospace Exploration Agency (JAXA). It uses the frequency of 10.7 GHz to estimate cloud-free SST with an inclined orbit that covers latitudes between 40°N and 40°S. The RMS difference with respect to buoys and infrared satellite instruments ranges from 0.5 °C to 0.7 °C (Wentz et al., 2000). The TMI time series was interpolated using the minimum-curvature method (Smith & Wessel, 1990) to fill the small data gaps due to data acquisition problems or contamination by heavy precipitation. Two distinct interpolation methods were intentionally used to avoid artificially increasing the cross-correlation because of the interpolator. The spatial and temporal resolutions of the SST time series were lowered by subsampling to match that of the altimetry data. The sea surface temperature anomalies (SSTA) were calculated with respect to the local subsampling to match that of the altimetry data. The sea surface temperature anomalies (SSTA) were calculated with respect to the local

4. Methodology

Offshore areas with a water depth of less than 1000 m were masked out because of the problems related to tidal aliasing in the altimeter data (Parke et al., 1998) (Fig. 1). The SSHA and SST maps were converted into longitude-time diagrams η(t, x) and θ(t, x), respectively, for each latitude. The propagating signals are dominated by Rossby waves that appear as a slanted striped pattern with a succession of crests and troughs propagating westward as time progresses.

After that, these longitude-time diagrams were filtered by a set of two-dimensional finite impulse response (FIR) filters to retain just the westward propagating signals separated from large-scale non-propagating and high frequency signals. Because of the filtering procedure, small islands were ignored and only the longitudinally continuous non-oceanic areas were considered for analysis. An advantage in these filters is that it is relatively easy to adjust their pass-bands to portray known geophysical processes. The set of filters were adjusted to decompose the original signals such that ηp = ηf + ηr + ηn (Fig. 2). ηr is the large-scale non-propagating signal that includes the seasonal and inter-annual variability. This component is obtained through a symmetric Gaussian bump that retains temporal scales ≥ 1 year and wavelengths of the same order of magnitude of the local basin width. ηf is the sum of all westward propagating signals with phase speeds similar to those of first mode baroclinic Rossby waves with periods between 1.5 months and 2 years (Polito & Liu, 2003). The apparent frequency overlap between the propagating and non-propagating signals with 1–2 years of period would be a problem if the temporal and zonal dimensions were filtered separately. However, the methodology applied here deals with the two dimensions simultaneously. Consequently, in the 2D zonal-temporal spectrogram the westward propagating signal occupies an area in the fourth quadrant, distinct and to the left of the non-propagating signal that lies over the zero-wavenumber axis, there is no overlap. Thus, the result of this process is a very broadband signal that propagates westward with a phase speed within 1/2 and 2 times the initial phase speeds used in Polito and Liu (2003). These authors used an extrapolation of the phase speeds from the standard theory (Chelton & Schlax, 1996) followed by one run of the FIR filter to obtain initial value of the phase speed. The propagating signals filter matrix is a tapered 2D cosine that forms the slanted pattern characteristic of waves. ηr is the residual obtained from ηp = ηf + ηr + ηn, containing short-scale, high frequency non-propagating signals shorter than 1.5 months, interpreted as noise. The same process with the same parameters was applied to SST data (similar definitions, with θ instead of η).

The 2D cross-correlation was calculated between ηr and θr to quantify a possible covariability between the longitude-time westward propagating components of SSHA and SST as a function of space and time lags, τx and τt, respectively. In the case of periodic phenomena, τx and τt would show a phase lag, and from that we can obtain an indication of the most probable physical process that connects the two variables. This method is particularly interesting because both Rossby waves and eddies were observed to have a spectral signature in SSHA and SST and the cross-correlation preserves the part of the spectrum that is common to both data series. This property

![Fig. 1. Spatial variance of the sea surface height fields in the Atlantic, as a proxy of the eddy kinetic energy. The solid lines are the isoline of 1000 m of depth and the dashed lines are the latitudinal limits used in this study, at 10.5°S and 35.5°S, respectively.](image-url)
applies to both wavenumber and frequency, thus the resulting phase speed is interpreted as the phase speed of the waves that are present simultaneously in the SSTA and SSHA records. Random red noise appears as a single central bump in the cross-correlation $C(\tau_{x}, \tau_{t})$, whose decay scale in terms of $\tau_{x}$ and $\tau_{t}$ quantifies the average length and duration of the anomalies. Conversely, periodic signals appear as a sequence of local maxima and minima with wavelength and period preserved by the cross-correlation calculation. Their phase speeds can be determined through the slope of the striped pattern observed in the longitude-time cross-correlation matrices.

A Radon transform analysis (Lim, 1990; Polito & Cornillon, 1997) was applied to the $C(\tau_{x}, \tau_{t})$ matrices to estimate the slope and therefore the phase speed of the Rossby waves. The Radon transform $\mathcal{R}(x_{i}, \alpha)$ rotates a pair of Cartesian axes $(x_{i}, t_{i})$ counterclockwise in relation to the fixed axes of the $(\tau_{x}, \tau_{t})$ matrix and associates this rotation to the angle $\alpha$. $\mathcal{R}(x_{i}, \alpha)$ is the integral of the $C(\tau_{x}, \tau_{t})$ coefficients along the $t_{i}$ axis, perpendicular to the $x_{i}$ axis defined as:

$$\mathcal{R}(x_{i}, \alpha) = \int_{-\infty}^{\infty} C(x_{i}, t_{i}) dt_{i}.$$

When the angle $\alpha$ is such that the high-correlation region is aligned with the $t_{i}$ axis, $\mathcal{R}(x_{i}, \alpha)$ is maximized and this particular angle $\alpha_{p}$ is the angle used to estimate the phase speed $c_{p}$, where $c_{p} = \tan(\alpha_{p})$. Period and wavelength are estimated by least squares fitting sinusoidal functions to the positive part of the largest maxima at zero zonal and temporal lags of $C(0, \tau_{t})$, $C(\tau_{x}, 0)$, respectively.

**4. Results**

The fraction of variance associated with non-propagating large-scale signals in the mid-latitudes South Atlantic region is $(22 \pm 6)\%$ for the SSHA and $(63 \pm 3)\%$ for the SSTA data (not shown). This large-scale non-propagating signal is composed mainly by the seasonal and interannual variabilities such as El-Niño/La-Niña. Although noticeable in the equatorial Atlantic, the latter prevails in the tropical Pacific. After this signal is removed, the variances associated with the westward propagating signals in the SSHA and SSTA are $(45 \pm 8)\%$ and $(28 \pm 7)\%$. The rest of the variance is dominated by small-scale, high frequency signals that are treated as noise.

This study describes the mid-latitude variability for each latitude between $10.5^\circ S$ and $35.5^\circ S$, and the 2D cross-correlation matrices (Fig. 3: 3rd columns) are presented every 5 degrees. These matrices can be described as a combination of two patterns: (1) a sequence of slanted stripes that span $\pm 20^\circ$ and one year or more, and (2) a single smooth bump at or near the origin. The first pattern is associated to periodic, deterministic signals, and the second to non-deterministic phenomena. Because of the different zonal-temporal scales and speeds, our interpretation is that these two patterns represent Rossby waves and mesoscale eddies, respectively. The shape of the central bump near the origin is often elongated with the same slope as the stripes. This is an indication that eddies at these latitudes move at approximately the phase speed of the waves. This hypothesis was also observed by Crawford et al. (2000), and suggests that eddy-like features can propagate embedded within a region of similar potential vorticity to the baroclinic Rossby waves, often with similar propagating speeds.

Hereafter, the analysis of Rossby waves and eddies signals takes advantage of the cross-correlation diagrams such as the ones presented in the Fig. 3: 3rd columns. The Rossby wave signals in the mid-latitudes South Atlantic Ocean show, on average, periodic maxima in correlation of approximately 0.3 in the longitude-time diagrams (Fig. 3). The strongest signals are observed between $10.5^\circ S$ and $15.5^\circ S$ (Fig. 3a, b), and are dominated by the annual frequency that extends to $19.5^\circ S$. Regions closer to the tropics often present deterministic signals that dominate the spectrum with a wavelength range from $10^{3}$ km up to the basin width and with frequencies from semiannual to biannual. Between $20.5^\circ S$ and $24.5^\circ S$ the annual Rossby waves are still the most prominent signal despite the increasing noise, more pronounced in the southern boundary (Fig. 3c). In this range of latitudes a transition between the prevalence of wave and eddy-like signals was observed. The Rossby wave energy peak moves from semi-annual to annual; the variability associated with Rossby waves decreases as a whole, while that associated with eddies increases. It is plausible that the poleward signal transition towards longer wave periods is related to the critical latitudes because, as the latitude increases, the ocean response to forcing at higher frequencies goes from Rossby waves to eddies. Latitudes higher than $25.5^\circ S$ presented a dominance of the biannual signal (see Fig. 3d–f). The Fig. 3d indicates that $25.5^\circ S$ is at the transition between annual and biannual dominance, and presents a rich spectral content that includes semiannual signals. Furthermore, it is at this latitude that the central bump, associated to eddies, becomes more prominent.
Fig. 3. Longitude-time diagrams of SSHA, SSTA and cross-correlation at (a) 10.5°S, (b) 15.5°S, (c) 20.5°S, (d) 25.5°S, (e) 30.5°S, and (f) 35.5°S in the Atlantic Ocean. The longitude-time diagram of SSHA in mm (left), SSTA in °C (middle) and SSHA-SSTA cross-correlation (right). The cross-correlation axes correspond to the lags in the longitude and time.
The latitude of 25.5°S corresponds to the northward limit of the South Atlantic Subtropical Gyre and to the Agulhas eddy corridor as described by Duncombe Rae et al. (1996) and Garzoli and Gordon (1996). This suggests that the prominence of eddies observed southward this latitude may be related to this high eddy variability region. The Agulhas retrofection at 35.5°S sheds rings into the South Atlantic, as part of the Agulhas leakage (Gordon, 1985), following a preferred northwesterly direction at approximately 25.5°S. These rings have an average diameter of about 300 km (Gordon & Haxby, 1990) and can be detected with the methodology used here. Additionally, Arhan et al. (1999) observed rings from the Agulhas retrofection with diameters of 200–500 km in the South Atlantic.

These mesoscale eddies are most evident south of 25.5°S, and are represented by the peak of correlation at the origin or slightly displaced from it, with values near 1 (Fig. 3d–f). The observed displacements of the cross-correlation peak from the origin suggest that the surface and the thermocline are slightly out of phase, in other words, the SSHA and SSTA data indicate that a complex dynamic process occurs between these layers. Meridional advection is presented as a plausible candidate because the SST gradients are more evident in this direction. An increase in correlation and a decrease in extension of the eddy signal at 34.5°S and 35.5°S is indicative of an increase of variability associated with eddies with propagation speeds significantly different from the \( c_p \) of Rossby waves (see Fig. 3f). In all latitudes, the maxima associated to eddies are at, or very close to the origin, therefore SSHA and SSTA are in phase. However, the striped patterns associated to Rossby waves have variable phase. Between 10.5°S and 15.5°S they show approximately zero phase and thus indicate vertical advection. The relatively shallow thermocline and weak meridional thermal gradient that characterize this latitudinal band reinforce this idea. To the south of these latitudes the phase of the relatively weaker striped patterns begins to vary and changes to 90 degrees around 19.5°S–25.5°S. This is clear in Fig. 3, panels e and f, where in all cases a wide blue band just grazes the origin, coming from below. This result is associated to horizontal advection by Rossby waves, granted the presence of a relatively stronger thermal gradient. Previous studies, such as Killworth et al. (2004); Gutknecht et al. (2010) have argued that the dominance of meridional advection explains the signature of planetary waves in chlorophyll-a concentration. Although this dominance is observed throughout their study region, Gutknecht et al. (2010) show a decrease contribution of the meridional advection of at least 10% and an increase of the vertical advection within the Subtropical Gyre. Our results corroborate this increase in the contribution of vertical advection in the gyre region, between 25.5°S and 35.5°S.

The spatial \( (d_i) \) and temporal \( (d_t) \) scales of these eddies are estimat-
ed from the cross-correlation \( C \), using an exponential decay, such that at the borders it has a correlation equal to 1/e times the maximum nearest to the origin. First, the single maximum nearest to the origin was isolated from within the one-dimensional vectors of the cross-correlation di-
agrams \( C(\tau,0) \) and \( C(0,\tau) \) extracted across the origin. After that, the value of maximum correlation was determined, as well as the pair of points approaching the 1/e scale. The distances between these pairs of points in \( C(\tau,0) \) and \( C(0,\tau) \) define \( \Delta d_i = \Delta d_0 \) and \( \Delta d_t = \Delta d_0 \) (Fig. 4), respectively. We have adopted the distance between two consecutive maxima or minima in the cross-correlation matrix as representative of the spatial and temporal scales of these non-random fluctuations. There are alternative methods to measure these scales such as the first zero-crossing or the 1/e decay scale. However, these definitions presume random or red noise and tend to underestimate the scales of the waves. Thus, \( d_i = \Delta d_0 / 2 \) and \( d_t = \Delta d_0 / 2 \), in kilometers and days, respectively. The determination of the temporal and spatial scales per se does not imply propagation. Nevertheless, an advective speed was calculated from \( c_p = d_i / d_t \) and the direction was imposed by an ad hoc negative sign justified by the observation of the slope of the patterns in the cross-correlation matrices.

The \( c_p \) values calculated by Radon transform are presented in Fig. 5 together with values from linear theory and other previous studies. Our average \( c_p \) values and those referenced to the other authors below correspond to the averaged \( c_p \) values for the same latitudes between 10.5°S and 35.5°S. Our \( c_p \) values show a predominantly positive bias in relation to the linear theory, with differences between 3 and 53% and an average of (32 ± 14)%. The theoretical values were calculated from a simplified linear theory used in Polito and Liu (2003). Considering that the phase speed for long first mode baroclinic Rossby waves is given by \( c_p = -f R_1/c_p \), \( c_p \) can be rewritten as

\[
c_p = -\frac{g H_1 \cos \phi}{2 R_1^2 \Omega \sin^2 \phi} - f B \cos \phi \sin \phi
\]

with \( B = -0,2 \) adjusted from the phase speed observed by Polito and Liu (2003).

Although the Radon transform method used in the present study was adapted from Polito and Liu (2003), our \( c_p \) values overestimated those found by the authors in an average difference of (24 ± 15)%.

The smallest difference occurred in 33.5°S and the largest in 21.5°S, 1% and 51%, respectively. Polito and Liu (2003) used a shorter time-series of SSHA and auto-correlation diagrams in their analysis. In addition, they separated the westward propagating signal in 4 different spectral bands, instead one band as presented here. We used their most energetic westward propagating signal on average (period of 1 year) to do these comparisons but according Polito and Liu (2003), next to 35.5°S the lower frequency signals are dominant in this region. Our speeds were on average (18 ± 19)% faster than Chelton and Schlax (1996), with a similar \( c_p \) at 10.5°S. The smallest and largest differences are 3% and 44% at 13.5°S and 23.5°S, respectively. These values were the nearest to our results in spite of our high standard deviations and the small number of observations by Chelton and Schlax (1996), a total of six estimates. Comparisons with \( c_p \) estimates found by Hill et al. (2000) and Challenor et al. (2004) showed average differences of (22 ± 14)% and (27 ± 15), respectively. In addition, to check the consistency of \( c_p \) calculated from the Radon transform over the cross-correlations, a manual measurement of the speeds was performed for each longitude-time diagram of SSHA, SSTA and cross-correlation for each latitude. The average difference between the values of \( c_p \) obtained from cross-correlations and that derived manually from the SSHA and cross-correlation (SSTA) were of 7% (5%), therefore considered statistically similar and not presented.

The \( c_p \) calculated by the spatial and temporal scales, \( d_i \) and \( d_t \), followed the same behavior described by the linear theory for Rossby waves (Fig. 6) with higher values poleward. A positive bias was observed when we compared our \( c_p \) to those presented by Chelton, Schlax, et al. (2011): speeds (32 ± 11)% faster on average. The propagating eddies are not persistent features, thus it is difficult to make precise comparisons regarding the propagation speed without synopticity. The methods used here are based in the observations of SSHA, SSTA and their cross-correlation matrices, and therefore provide statistical information that includes all eddies that are present in both datasets regardless of their shape or internal dynamics. Chelton, Schlax, et al. (2011) tracked individual eddies, identified by the Okubo-Weiss parameter and applied the least squares fit on the latitudes of the centroids of each eddy to estimate its propagation speed. Comparisons between our estimated \( c_p \) and \( c_p \) shows a striking similarity within the studied domain and a slight bias toward eddies faster than waves in 54% of the studied latitudes. 70% of all \( c_p \) estimates showed absolute differences of less than 11% in relation to \( c_p \), and 40% showed differences of less than 5%. The higher differences were observed in 24.5°S and 25.5°S where the eddy speeds are ~76% and ~79% smaller than those of the Rossby waves, respectively. The average absolute difference between \( c_p \) and \( c_p \) was (15 ± 20)%. These results corroborate Chelton, Schlax, et al. (2011) in the sense that there are westward propagating eddies with propagation speeds similar to the \( c_p \) of baroclinic Rossby wave in most of the mid latitudes. However, planetary waves (1) are the clearest signal in the cross-correlations in the equatorward half of the

\[
\frac{\beta}{C_1} = \frac{H_1}{R_1^2} \cos \phi
\]
our domain (Fig. 3a–c), and (2) seem to interact with the eddies, conditioning their propagation speed (Fig. 3d–f).

5. Summary

An analysis of variance in the longitude-time diagrams shows that the large-scale non-propagating signal explains on average almost three times more fractional variability in the SST when compared with the SSH. It is higher northward of 20.5°S in the SSH data and the opposite is observed in the SST. After its removal, the westward propagating signal in the SSH explains on average (45 ± 8)% of the variability, versus (28 ± 7)% of the SST. The dominance of the seasonality in the SSTA record is well known and its quantification lends credibility to the methodology used here.

The use of the cross-correlation between SSHA and SSTA is a new method to analyze westward propagating signals and has the capability to distinguish between signals associated with baroclinic Rossby waves and mesoscale eddies. Furthermore, the cross-correlations corroborate the hypothesis that the variability associated with SSHA and SSTA data results from a balance between vertical and horizontal advection. The vertical mechanism is related to perturbations in the SSHA field by passage of the Rossby waves and an advection of a vertical temperature gradient; here we have relatively large gradients and a small velocity associated with the ageostrophic divergence of mass. In the second case, SSHA variations and vertical displacements of the thermocline generate a meridional flux that, associated with a background meridional SST gradient, can promote SSTA anomalies by horizontal advection; here we have relatively small gradients and a large velocity associated with the geostrophic motion. The distinction between these mechanisms is observed by the phase lag between SSHA and SSTA: 90° in the first case and 0° in the second.

The latitudes between 10.5° S and 24.5°S showed a dominance of the annual Rossby waves with predominance of vertical advection between 10.5°S and 19.5°S and horizontal advection from 20.5°S to 24.5°S.
24.5°S. A transition region from annual to biannual signals is clearly observed poleward of 25.5°S. This comes together with an increasing eddy signal and a phase lag indicative of growing vertical advection that extends up to approximately 34.5°S, although the horizontal advection is still the dominant mechanism. In this context, 25.5°S can be considered the transition latitude between the dominance of wave and eddy signals, where it is possible to observe that the variability is dominated by eddies poleward of it (Fig. 3d–f). This is related to the region of high eddy variability known as Agulhas eddy corridor (Duncombe Rae et al., 1996; Garzoli & Gordon, 1996) that extends between 25.5°S and 35.5°S in the South Atlantic. Thus, based on Fig. 3d–f, we suggest that there are eddies traveling superimposed to Rossby waves similar to what was observed by Crawford et al. (2000) in the Alaskan Stream region. The decrease in extension of the cross-correlation peak at 34.5°S–35.5°S matches with a dominance of vertical advection and suggests a variability dominated by eddies with propagation speeds different from that of the Rossby waves.

Our $c_p$ estimates were most often faster than those suggested by the simplified linear theory presented by Polito and Liu (2003), with an average absolute difference of 32% and in consonance with what was previously observed by Chelton and Schlax (1996), Killworth et al. (1997), Killworth and Blundell (2003a) and others. This study and Polito and Liu (2003) used the same implementation of the Radon transform method to estimate the phase speed of Rossby waves. However, the application of FIR filters was conducted differently and the interpolation method and time series length are different. However, the most important distinction is that we performed a simultaneous analysis of SSHA and SSTA. At first, the decomposition of the Rossby waves signal in five different spectral bands used by Polito and Liu (2003) was taught as the most important factor in the determination of discrepant phase speed estimates. However, Appendix A shows that the use of a single broadband instead of narrower bands to estimate $c_p$ doesn’t necessarily result in faster speeds. The decomposition of the westward propagating signal in narrower bands cannot be considered the main reason for the differences between our estimates and those found by Polito and Liu (2003). In terms of methodology, our study is similar to Chelton and Schlax (1996), since the westward propagating spectrum was taken as a broadband process. The $c_p$ values presented by them were the nearest to our estimates, with an average difference of 18% and differences smaller than 12% in about 70% of the latitudes observed by these authors.

From the test results presented in Table A.1 we can say that the broadband results signal in phase speed estimate that is often equal to the phase speed associated to one of the narrow bands in 50% of the tests. The phase speed of the broadband is the fastest in 10% of the cases. In the other 90% of the cases, the phase speed from the broadband signal is either equal to that obtained from one of the bands, or similar to the mean phase speed. Thus we cannot say that the difference in phase speeds is simply a consequence of using broadband signals. There are just a few examples (10% of the cases) where significant differences are observed between $c_p$ values from the same narrow spectral band collected in different dates and regions. Thus, the length of the time series and the exact areas where the phase speeds were calculated are unlikely to be the major cause of the observed phase speed discrepancies.

The major methodological difference is that this study uses the SSHA and SSTA simultaneously, via cross-correlation, and Polito and Liu (2003) used only the SSHA auto-correlation. The cross-correlation selects the part of the 2D spectrum that is present in both data sets. The input signal is different, thus we should expect different results. Regions where the $c_p$s are significantly different are interpreted as regions where the surface processes that control the SSTA are not phase-locked with the pycnocline level processes that control the SSHA.

The $c_p$ estimates presented a slight positive trend in relation to estimates by Chelton, Schlax, et al. (2011), around 32% faster. Here it is important to point out that our estimates are based only in the observations of the cross-correlation matrices, while Chelton, Schlax, et al. (2011) takes advantage of the kinematics of these eddies to determine their position and speed. Therefore, our results take into account, in a statistical sense, all eddies within the region that are present in both datasets, regardless of shape or details of their circulation. Our $c_p$ estimates showed a significant similarity with $c_p$, except in 24.5°S and 25.5°S where the eddies are – 76% and – 79% slower than the Rossby waves, respectively. Differences between $c_p$ and $c_e$ were less than 11% in 70% of the studied latitudes, and suggests that the proposed methodology is sufficiently robust to distinguish among different propagating signals, at least in mid latitudes. Both $c_p$ and $c_e$ were estimated from the same longitude-time diagram, thus their differences are not caused by methodological inconsistencies. We conjecture that this increase in the eddy speeds can be caused by the advection by surface currents in this region. The main point is that meso- to large-scale eddies and long first mode baroclinic Rossby waves coexist and we were able to distinguish them through the cross-correlation analysis of SSHA and SSTA data.

This simultaneous analysis of SSHA and SSTA has shown consistent evidences that in mid-latitudes there are Rossby waves and eddies. They may propagate together or not. Depending on regional conditions and on the latitude, this combination of waves and eddies assumes different proportions and are linked to meridional and vertical advective processes.

Acknowledgments

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Appendix A. Estimates of phase speeds from a broadband vs. narrow bands

The objective of the test below is to check if the $c_p$ estimated from a broadband signal used in this study is consistently faster than those found by the decomposition of this signal in narrow bands used by Polito and Liu (2003). We used Topex/Jason records at 27.5°N and 27.5°S in the Pacific and Atlantic ocean and at 27.5°S in the Indian ocean to perform this test. The same latitude was purposely selected by the reviewers in this manuscript. This work was supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) under grant 140713/2006-9.

The Table A.1 shows that the broadband signal is not necessarily faster than the decomposed signals. The area 1 in the Pacific ocean was the only one to presented faster $c_p$ values in the broadband signal. Variations in $c_p$ seem to be related to regional and temporal variations (e.g., intensity of the ocean currents, regional wind-driven
forcing, and variability of the density fields). There are no variations in \( c_p \) related to \( \beta \)-effect and topography since we have maintained the absolute value of latitudes in these estimates and each ocean basin was checked in two different periods. We concluded that the difference between our \( c_p \) estimates and those found in the literature presented in Sections 4 and 5 cannot be attributed simply to the bandwidth of the input signal.

Another difference regards details of the implementation of the Radon transform. For each latitude and basin, Polito and Liu (2003) calculated the RT of the auto-correlation in as many sub-areas measured approximately 1 wavelength by 1 period as possible. For each sub-area they got 1 value of \( c_p \). From each set of \( c_p \) they eliminated outliers more than 2 standard deviation from the mean and areas with low amplitude. Their presented values are the mean of the remaining \( c_p \) values. Here, the whole longitude-time diagram was used to obtain the cross-correlation and from that a single \( c_p \) is calculated. This \( c_p \) is taken as representative of the mean \( c_p \).

However, the major difference is that this study uses the SSHA and SSTA simultaneously, via cross-correlation, and Polito and Liu (2003) used only the SSHA auto-correlation. The cross-correlation selects the part of the 2D spectrum that is present in both data sets. The input signal is different, thus we should expect different results. Regions where the \( c_p \) is significantly different are interpreted as regions where the surface processes that control the SSTa are not phase-locked with pycnocline level processes that control the SSHA.

### References


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### Table A.1

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