Two-mirror telescope design with third-order coma insensitive to decenter misalignment
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Lucimara Cristina Nakata Scaduto,1,2,* Jose Sasian,3 Mario Antonio Stefani,2 and Jarbas Caiado de Castro Neto1,2

1Instituto de Física de São Carlos, Universidade de São Paulo (IFSC-USP), Av. Trabalhador São-carlense, 400, CEP:13560-970, São Carlos, SP, Brazil
2Department of Research and Development, Opto Eletrônica S.A., R. Joaquim A.R. de Souza, 1041, CEP:13563-330, São Carlos, SP, Brazil
3College of Optical Sciences, University of Arizona, 1630 East University Boulevard, Tucson, Arizona 85721, USA

*lucimara@ifsc.usp.br

Abstract: Misalignments always occur in real optical systems. These misalignments do not generate new aberration forms, but they change the aberration field dependence. Two-mirror telescopes have been used in several applications. We analyze a two-mirror telescope configuration that has negligible sensitivity to decenter misalignments. By applying the wave aberration theory for plane-symmetric optical systems it is shown that the asphericity in the secondary mirror, if properly chosen, can compensate for any decenter perturbation allowing third-order coma unchanged across the field of view. For any two-mirror system it is possible to find a configuration in which decenter misalignments do not generate field-uniform coma.

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References and links

1. Introduction

Two-mirror telescopes play an important role in astronomy and other applications. The Cassegrain type telescope is a two-mirror imaging system with positive focal length. The classical Cassegrain configuration is obtained if both mirrors are independently corrected for spherical aberration.

A two-mirror telescope corrected for both spherical aberration and linear coma, that is aplanatic, is the Ritchey-Chrétien (RC) configuration. This system has hyperbolic primary and secondary mirrors [1].

For Cassegrain telescopes, the extent of the field of view is limited by coma aberration with a linear field dependence. Astigmatism aberration is usually negligible for small fields and has a quadratic field dependence [2].

Whereas the Cassegrain field of view is limited by uncorrected third-order coma, the RC field of view is limited by uncorrected third-order astigmatism [3].

Two types of misalignments that may occur in an optical system are considered in this study: tilt and decenter. Tilted elements occur when the optical axis of some element are not parallel to the optical axis of the system and decentered elements occur when their vertex is displaced from the optical axis of the system.

In axially symmetric systems, the third-order aberrations that can be present are spherical aberration, linear coma, quadratic astigmatism, field curvature and cubic distortion. When the axial symmetry is broken due to misalignments, different field-dependent aberrations may occur [4].

The field of spherical aberration is not affected by misalignments. In an aligned system, coma may have a linear field dependence or may be corrected over the field. Uniform coma is generated in the presence of misalignments. If linear and uniform coma are present in the system, the third-order zero-coma point (node) is not located in the center of the field. Misalignments will generate binodal astigmatism, that is two points in the field of view with zero-astigmatism [4, 5].

For two-mirror telescopes, we can consider the primary mirror as a reference. Misalignments of the secondary mirror generate, among other aberrations, linear field-dependent astigmatism, which manifests as binodal astigmatism, and uniform coma. These aberrations may occur in plane-symmetric systems, as explained by Sasian [4]. For this type of systems third-order uniform coma is the most important misalignment-induced aberration [5].

Schmid [3] described the effects of misalignments on the field dependence of the third-order aberration fields of two-mirror astronomical telescopes by applying the so called nodal aberration theory [6]. Schmid provides a complete description of the aberration content and distribution over the field of view.

Several authors [3, 5–9] have discussed the alignment of a two-mirror optical system and different techniques have been developed to account for third-order coma and astigmatism correction.

Some alignment methods based on the elimination of on-axis coma have been applied for two-mirror systems over the years. For slow f-number, small field of view Cassegrain telescopes, such alignment methods can be sufficient. However, for aplanatic wide-angle systems, as in RC telescopes, measurements of coma and astigmatism only on-axis do not seem to be sufficient since astigmatism may have a second node at an off-axis field point. This fact permits the systems to remain misaligned even if the telescope is aberration-free on-axis [3, 7]. To correct for astigmatism, measurements can be performed in off-axis positions, as proposed by McLeod [9].

Some methods for designing two-mirror systems with reduced sensitivity to misalignments have also been previously proposed [10, 11].
This paper discusses a two-mirror, less-sensitive, telescope configuration, previously presented by Wilson [11], that is corrected for spherical aberration, and which is insensitive to decenter misalignments for field-uniform coma. Based on the wave aberration theory of plane-symmetric optical systems [4], which restricts surface tilts to one dimension with the gain of simplicity and ease of understanding, it is shown that for this configuration no uniform coma is generated by decenter misalignments. This feature makes the resulting RMS wave error to remain low even with considerable values of decenter in the secondary mirror. Such a telescope configuration can be useful for some applications.

Although the less-sensitive configuration is not aplanatic, i.e. it has linear field-dependent coma, it is insensitive to misalignment as not to generate uniform coma. The third-order linear coma node does not change location in the presence of secondary mirror decenters and remains the same for both aligned and misaligned states. Some amount of uniform astigmatism may occur. The linear coma present in the less-sensitive two-mirror configuration can be corrected in a relay system to couple a detector or science instruments.

Tilt perturbations of the secondary mirror generate uniform coma and change the third-order linear coma node position. Tilt misalignment can be mitigated if a Serrurier truss is used to mount the mirrors.

Correcting the uniform coma by changing only the tilt of the secondary mirror makes the alignment procedure of this specific configuration simpler, in comparison to other configurations.

The less-sensitive configuration will allow for a smaller useable field of view in comparison to the classic Cassegrain or a Ritchey-Chrétien configuration. However, as pointed above, the linear coma can be corrected elsewhere, when necessary, leading to an overall less-sensitive system.

The less-sensitive two mirror configuration is based on the proper choice of the surface conic constants. Since it is possible to find the conic constant of one of the mirrors based on the conic constant of the other, the appropriate choice of the conic constant of the secondary provides a configuration that is less affected by decenter misalignments [11].

This “special choice” of the conic constant of the secondary mirror depends on three basic constructional parameters: the effective focal length (EFL) of the two-mirror system, the distance between the mirrors and the back focal length (BFL). This paper reports on the study of two-mirror systems that have a concave primary and a convex secondary.

In our analysis we assume that the primary mirror figure corrects for high orders of spherical aberration as this is a standard assumption and practice.

The paper is organized as follows: Section 2 briefly describes two-mirror telescopes; Section 3 compares the sensitivity due to misalignments of a classical Cassegrain telescope, a Ritchey-Chrétien telescope and a similar configuration designed for the less-sensitive condition; Section 4 describes the aberrations generated in this system based on Sasian [4] approach for plane-symmetric systems; finally Section 5 concludes the paper.

2. Two-mirror telescopes

By determining the three parameters, effective focal length \( f \), back focal distance from the secondary mirror vertex to system focus \( B \) and separation between the mirror vertices \( d \), the radius of curvature \( R_p \) of the primary mirror \( M_p \) and the radius of curvature \( R_s \) of the secondary mirror \( M_s \) can be written as [1]:

\[
R_p = \frac{2df}{B-f}, \quad 1
\]

\[
R_s = \frac{2dB}{B+d-f}. \quad 2
\]
Figure 1 shows the two-mirror configuration considered in our study and the parameters used. The conic constant of the primary mirror \( k_p \) can be adjusted according to the conic constant chosen for the secondary \( k_s \) for a two-mirror system corrected for third-order spherical aberration:

\[
k_p = \left( \frac{B}{f} \right) \frac{(B+d-f)}{(B-f)^3} \left[ (B+d-f)^{2} k_s + (f+d-B)^2 \right] - 1 .
\]

The classical Cassegrain or Gregorian configuration is obtained if both mirrors are independently corrected for third-order spherical aberration, which leads to the following conic constants:

\[
k_p = R_p \left( \frac{f-B}{8d^3 f} \right)^3 ,
\]

\[
k_s = R_s \left( \frac{f-d-B}{8d^3 B^3} \right) .
\]

The Ritchey-Chrétien telescope (RC), which is also corrected for third-order coma, has a specific secondary conic constant given by

\[
k_s = R_s \left[ \frac{2f(B-f)^2 + (f-d-B)(f+d-B)(d-f-B)}{8d^3 B^3} \right] .
\]

### 3. Sensitivity due to misalignments

#### 3.1 Decenter and tilt-induced aberrations – perturbation analysis

The sensitivity of some two-mirror configurations due to decenter and tilt misalignments was evaluated for systems with EFL=3750mm, BFL=1200mm, d=800mm, and different conic constants. Equations (1) and (2) provide the radius of curvature of the concave primary and convex secondary mirror for these parameters, respectively \( R_p = -2352.94 \text{mm} \) and \( R_s = -1097.14 \text{mm} \). By specifying the conic constant of the primary mirror \( k_p \) it is possible to find the conic constant of the secondary mirror \( k_s \) using Eq. (3), which corrects for third-order spherical aberration (Table 1).

Perturbing the secondary mirror by inserting different amounts of tilt or decenter, permits evaluating how the on-axis RMS wavefront error, referenced to the centroid, is affected by decenter and tilt misalignments (Fig. 2).
Table 1. Conic constants of the primary and secondary mirrors for the different configurations.

<table>
<thead>
<tr>
<th>( k_p )</th>
<th>( k_s )</th>
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<tbody>
<tr>
<td>-1.00</td>
<td>-3.66</td>
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<tr>
<td>-0.90</td>
<td>-2.70</td>
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<tr>
<td>-0.80</td>
<td>-1.73</td>
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<td>-0.76</td>
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<td>-0.36</td>
<td>2.52</td>
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<tr>
<td>-0.34</td>
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<td>-0.30</td>
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</tr>
<tr>
<td>-0.20</td>
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<tr>
<td>-0.10</td>
<td>5.04</td>
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<tr>
<td>0.00</td>
<td>6.00</td>
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</table>

The different configurations exhibit different sensitivities due to decenter and one specific configuration seems to be the least affected. The sensitivity of these different configurations is not significantly affected by tilt.

These results indicate that there is a combination of conic constants that softens the effects of decenter-induced aberrations.

3.2 Comparison of sensitivities – Cassegrain, Ritchey-Chrétien and less-sensitive telescopes

A comparison of the on-axis RMS wavefront error due to misalignments of three similar configurations of two-mirror telescope, i.e., same EFL, BFL and separation between mirrors, was performed. The conic constants of the mirrors are different in these configurations and have a strong influence on the alignment sensitivity of the system.
The three configurations chosen were the classical Cassegrain telescope, in which the primary mirror is parabolic in shape and both mirrors are corrected for spherical aberration, the Ritchey-Chrétien (RC) telescope, which is corrected for spherical aberration and linear-coma, and a third system, which presents low sensitivity to decenter perturbations (LS) [11].

The STOP was located at the primary mirror and the main parameters of the systems are EFL=3750mm, BFL=1200mm, and d=800mm. Table 2 exhibits the conic constants for the three configurations and the residual linear coma ($W_{1300}$) and quadratic astigmatism ($W_{2200}$) for the aligned state for a ± 0.25° field of view.

By considering the primary mirror as the reference of the system, the sensitivity of the different configurations was analyzed with respect to the misalignment of the secondary mirror. Tilt (Fig. 3) and decenter (Fig. 4) were taken into account in this analysis. The misalignments were applied to the x-axis.

For a comparison of the three systems, the Zernike Fringe Polynomial coefficients ($Z_5$, $Z_6$, $Z_7$, and $Z_8$) were taken on-axis for the aligned and perturbed states. Coefficients $Z_5$ and $Z_8$ are associated with astigmatism and coefficients $Z_7$ and $Z_8$ are associated to coma aberrations.

Coefficients $Z_5$ and $Z_8$ are the most affected by tilt perturbations in the x-axis. The three configurations exhibit similar behavior (Fig. 3). Tilt strongly affects coma and astigmatism fields of all configurations and the on-axis values of these aberrations are no longer zero.
Coefficient $Z_7$ is the most affected by decenter perturbations in the x-axis. Figure 4 shows that the LS configuration is not affected by lateral decentering coma.

This specific configuration led to a system with low sensitivity to decentering misalignments as it is free from the most important aberration generated by lateral decenter, i.e., field-uniform coma. The absence of field-uniform coma in the misaligned state makes the third-order coma across the field remain the same as for the aligned state.

Interestingly, the less-sensitive configuration has the least amount of quadratic astigmatism; about a factor of seven less than the classical Cassegrain, and a factor of eight less than the Ritchey-Chretien.

Fig. 3. Graphics of the Zernike Fringe Polynomial coefficients: A) $Z_3$; B) $Z_6$; C) $Z_7$; D) $Z_9$ and E) RMS referenced to the centroid (waves, $\lambda = 0.55\mu m$) as a function of tilt perturbation for the classical Cassegrain, the RC and the LS configurations.

4. Aberration theory approach

Seidel studied aberrations in rotationally symmetric optical systems and provided specific sums to determine the primary aberration coefficients [1, 11]. When there are tilted or decentered components in the system, it becomes non-rotationally symmetric. Several authors have described the aberrations in non-rotationally symmetric systems. R. Buchroeder, in 1976, studied tilted component systems and his work resulted in the insight that the aberrations at the image plane of a non-symmetric or misaligned optical system is still a sum of the individual surface contributions, although the individual contributions no longer have a common center on-axis.

R. Shack and K. Thompson developed Buchroeder’s insights by writing the aberration function in vector form to account for component tilt, and allowing some seldom seen vector operations such as vector multiplication. This formalism is the appropriate tool to combine the aberrations of tilted components and resulted in the concept of aberration fields and...
nodes. In non-rotationally symmetric systems, no new aberrations are created, but new field dependences occur. The order of these new field dependences go down from the power that exists in the rotationally symmetric system [3, 8].

R. Buchroeder, R. Shack and K. Thompson explained that when the elements in a system are misaligned, the contribution from each surface in the system to the aberration field remains rotationally symmetric about some point in the field. For an aspheric surface, due to its unique axis, the shift of the field center of the spherical and aspheric parts will be different, but for both components the symmetrical nature of the aberration to its shifted center is maintained [3, 5–7].

![Fig. 4. Graphics of the Zernike Fringe Polynomial coefficients: A) Z_{45}; B) Z_{65}; C) Z_{75}; D) Z_{85} and E) RMS referenced to the centroid (waves, \( \lambda = 0.55\mu m \)) as a function of decenter perturbation for the classical Cassegrain, the RC and the LS configurations.](image)

The so called Nodal Aberration Theory was introduced by Thompson to describe the locations in the field where a specific aberration sum to zero [6, 8, 12].

Sasian [4] developed a useful theory for plane-symmetric optical systems (Appendix A) and noted that line nodes can also exist. He presented an aberration function to describe the image position and size, image defects and specific aberration fields that occur in axially symmetric, double-plane symmetric and plane-symmetric optical systems. The occurrences of specific aberration fields in each of these systems were analyzed. This approach is applied here for the understanding of the two-mirror less-sensitive system.

The graphics above show that for the classical Cassegrain and RC configurations, both tilt and decenter of the secondary mirror cause total coma and astigmatism aberration fields to change when compared with the aligned state, in the case of the less-sensitive system, tilt affects substantially the coma aberration field.
When decenter occurs in the secondary mirror of an LS system, the third-order coma aberration field is not changed, i.e., the linear coma present in the aligned system state remains in the decentered system state, and the total coma keeps the same field dependence and magnitude. Therefore decenter misalignment does not generate uniform coma and the coefficient $W_{03001}$ is zero.

For an aspheric surface, the total uniform coma coefficient can be considered a summation of a spherical “base” surface and an aspheric “cap” contributions.

### 4.1 Spherical contribution to uniform coma

According to Sasian [4], for tilted components, the spherical contribution to the uniform coma coefficient in a two-mirror system is given by (Appendix A)

\[
W_{03001}^s = \left[ J_{tt}^p + J_{tt}^s \right] = -\frac{1}{2} \left[ n_p \sin(I_p) A_p \Delta \left( \frac{u}{n_p} \right) x_p + n_i \sin(I_i) A_i \Delta \left( \frac{u}{n_i} \right) x_i \right].
\]

(7)

For the two-mirror telescope:

\[
A_p = \frac{m}{2f} x_p \cos(I_p),
\]

(8)

\[
A_s = \frac{m}{f} \left[ 1 - \frac{(m+1)}{2m} \cos(I_i) \right] x_p,
\]

(9)

\[
\Delta \left( \frac{u}{n} \right) = \frac{m}{f} x_p,
\]

(10)

\[
\Delta \left( \frac{u}{n} \right) = -\frac{m}{f} \left( 1 + \frac{1}{m} \right) x_p.
\]

(11)

We have:

\[
W_{03001}^s = -\frac{1}{2} \left[ \frac{m}{2f} \sin(I_p) \cos(I_p) + \frac{Bm}{f^2} \sin(I_i) \cos(I_i) \right] x_p^3,
\]

where $m = f / f_1^*$, in which $f_1^*$ is the effective focal distance of the primary mirror.

Since the primary mirror is the reference, i.e., no decenter or tilt is imposed on it, we have $I_p = 0$, which leads to $\sin(I_p) = 0$. Therefore only the secondary mirror contributes to the spherical part of the uniform coma coefficient:

\[
W_{03001}^s = -\frac{Bm}{2f^3} \sin(I_i) (m+1) \left[ 1 - \frac{(m+1)}{2m} \cos(I_i) \right] x_p^3.
\]

(13)

Equation (13) can be used to describe a decenter misalignment of the secondary mirror. For a displacement of the vertex of a spherical surface (Fig. 5), the effect on the optical axis can be considered as a displacement $\delta_{\text{sphere}}$ of the center of curvature of this surface, which is similar to a tilt $(I_i)$ of the surface about its vertex. By admitting that the longitudinal displacement of the surface in the OAR intersection $(\rho_y)$ is negligible, $\delta_{\text{sphere}}$ can be written in terms of the tilt angle $(I_i)$ as

\[
\delta_{\text{sphere}} = \frac{m}{f} (m+1) \cos(I_i) x_p.
\]
\[
\sin(I_s) = \frac{\delta_{\text{sphere}}}{R_s}.
\] (14)

As \(I_s\) is small, another valid approximation in this case is \(\cos(I_s) = 1\) and the spherical contribution to uniform coma for decenter misalignments can be described by

\[
W_{03001}^s = -\frac{1}{8} \left( \frac{x_p}{f} \right)^3 (m+1)^2 (m-1) \delta_{\text{sphere}}.
\] (15)

Fig. 5. Decenter of a spherical surface.

4.2 Aspheric cap contribution to uniform coma

According to Sasian [4] the aspheric contribution of the tilted surface to uniform coma aberration is given by (Appendix A)

\[
W_{03001}^s = \beta \Delta [n \cos(I_s)] x^3.
\] (16)

The conic constant of the secondary exerts no influence on coma when tilt is present in the system. This fact was also pointed out by Wilson [11], and this explains why the RMS variation due to tilt is not strongly affected by the different conic constants combinations (Fig. 2).

Concerning tilt perturbation, the sensitivity depends only on the radius of curvature of the surface, which has been set identical for the three analyzed systems resulting in identical sensitivity.

For a decenter perturbation of the secondary mirror, the aspheric contribution to uniform coma aberration must be considered and can be derived from the sag of the surface:

\[
Z = \frac{\left( x^2 + y^2 \right)}{2R} + \frac{(k+1) \left( x^2 + y^2 \right)^2}{8R^3},
\] (17)

The aspheric cap contribution to the secondary mirror sag is given by

\[
Z_{\text{sphere}} = k_s \frac{\left( x^2 + y^2 \right)^2}{8R^3}.
\] (18)

The wavefront from the secondary is
\[ W^* = k \frac{(x^2 + y^2)^2}{8R^3} \Delta(n) . \]  

(19)

If the beam is decentered by \( \delta_{\text{asphere}} \) in the x-axis the wavefront becomes

\[ W^*_{\text{decenter}} = k \frac{\left((x + \delta_{\text{asphere}})^2 + y^2\right)^2}{8R^3} \Delta(n) . \]  

(20)

Therefore, the aspheric contribution to the third-order uniform coma for the two-mirror system with the entrance pupil at the primary mirror is

\[ W^*_{03001} = k \frac{4x}{8} \delta_{\text{asphere}} \Delta(n) . \]  

(21)

We have then

\[ W^*_{03001} = \frac{1}{8} \left( \frac{x}{f} \right)^3 (m+1)^2 k_x \delta_{\text{asphere}} . \]  

(22)

Equation (22) above gives the aspheric contribution of the secondary mirror to the uniform coma when decenter is present.

4.3 Uniform coma aberration coefficient for decentered two-mirror systems

By summing the spherical and aspheric contributions, the total uniform coma coefficient is given by

\[ W^*_{03001} = -\frac{1}{8} \left( \frac{x}{f} \right)^3 (m+1)^2 \left[ (m-1) \delta_{\text{asphere}} - (m+1) k_x \delta_{\text{asphere}} \right] . \]  

(23)

When the same misalignment of the spherical surface and aspheric cap is considered, the total uniform coma coefficient is

\[ W^*_{03001} = -\frac{1}{8} \left( \frac{x}{f} \right)^3 (m+1)^2 \left[ (m-1) - (m+1) k_x \right] \delta . \]  

(24)

In a less-sensitive system, the uniform coma coefficient must be zero so that the field of coma does not change as a function of decenter misalignments of the secondary mirror

\[ \left[ (m-1) - (m+1) k_x \right] = 0, \]  

(25)

which gives

\[ k_x = \frac{(m-1)}{(m+1)}. \]  

(26)

As \( m = \frac{f}{f_1} = \frac{(B - f)}{d} \), we can also write

\[ k_x = \frac{B - f - d}{B - f + d}. \]  

(27)
Equation (27) above allows the calculation of $k_s$ as a function of the EFL, BLF and mirrors distance, which leads to a two-mirror system configuration with low sensitivity to misalignments since this conic constant makes decenter-induced field-uniform coma remain zero.

4.4 The coma-free point

In two-mirror systems there is a point in the optical axis for which coma generated by translation and rotation of the secondary mirror cancel each other \cite{11, 13}. This point is called the coma-free point (measure from $M_s$ vertex) and is one of two neutral points present in a two-mirror telescope system. Rotations of the secondary mirror about this point do not cause coma aberration.

This point can be used for the alignment of two-mirror systems, since it enables the correction of astigmatism in the system without introducing any coma \cite{8, 9, 11}.

By making equal to zero the equation obtained by Wilson \cite{11} for the total coma (including rotational and translational contributions) the coma-free point $Z_{\text{CFP}}$ can be found in a two-mirror system by the following equation

$$Z_{\text{CFP}} = \frac{2f(m-1)R_4}{(m+1)^2 \left[ \frac{(m-1)}{(m+1)} - k_s \right]}$$

where $R_4 = \left(1 + \frac{2d}{R_p}\right)$.

Ren et al. \cite{14} deduced the coma-free point by a vector approach and found the same equation.

When the conic constant $k_s$ of the less-sensitive system is substituted in the Eq. (28), we find $Z_{\text{CFP}} = \infty$, indicating that the pivot point for coma is located at infinity \cite{11}. This represents a lateral movement of the secondary rather than a rotation.

Figure 6 exhibits the behavior of the location of the coma-free point $Z_{\text{CFP}}$ as a function of the conic constant of the secondary mirror for two different systems. The function of $Z_{\text{CFP}}$ has a point of singularity in a specific conic constant value, which corresponds to a decenter less-sensitive system.
If there is an error $\Delta k_s$ in the conic constant of the secondary mirror, then a residual uniform coma takes place which is:

$$W_{0001}^{\text{residual}} = \frac{1}{8} \left( \frac{x}{f} \right)^3 (m+1) \left[ -(m+1) \Delta k_s \right] \delta_{\text{sphere}}.$$

Equation (29)

Figure 7 shows coefficient $Z_7$ versus decenter misalignment of the LS configuration when there is an error of 0.004, 0.03 and 0.1 in the secondary mirror conic constant $k_s$.

5. Conclusions

When misalignments occur in a system, no new aberration forms are created, but new field dependences arise. In a two-mirror system, coma aberration is the most affected by tilt or decenter errors of the secondary mirror. This follows from the fact that uniform coma is linear with respect to mirror tilt. Uniform astigmatism depends on the square of the mirror tilt.
In the aligned state the system may exhibit either linear coma or no coma, as in the RC telescope. When the axial symmetry is broken by a misalignment, uniform coma is generated. A specific combination of conic constants leads to a system that is less affected by decenter perturbation [11]. The approach used in our study is based on the wave aberration function for a plane-symmetric optical system. It is shown that for this system, no field-uniform coma is generated and the system shows low sensitivity due to secondary mirror decenter misalignments. In this configuration, the aspheric cap contribution to uniform coma and the spherical base surface contribution cancel each other and the resultant uniform coma aberration is zero.

The less sensitive system configuration does not exhibit uniform coma in its decentered state, but suffers from linear field-dependent coma with the third-order linear coma node on-axis. Nevertheless a negligible amount of uniform astigmatism and some linear astigmatism is generated for the decentered state. However, the less-sensitive configuration has the least amount of quadratic astigmatism; about a factor of seven less than the classical Cassegrain, and a factor of eight less than the Ritchey-Chretien.

Despite the low sensitivity due to decenter perturbation, the less-sensitive configuration is still affected by tilt perturbations, which generate uniform coma and change the third-order linear coma node position. A Serrurier truss can be used with effectiveness to mitigate this tilt misalignment error. The third-order linear coma can be corrected in a much smaller relay or stage optics downstream to result in an overall less-sensitive telescope system. Trading off one feature by another in two-mirror telescopes is becoming a subject of current interest. For example, a two-mirror telescope [15] configuration that has practical value produces significant amounts of linear-coma, but no diffraction rings. The linear coma is corrected elsewhere in the optical train.

Overall, this system solution is of theoretical interest and practical applications.

Appendix

A – Wave aberration theory for plane-symmetric optical systems

In the wave aberration theory approach for plane-symmetric optical systems [4] the optical axis ray (OAR) is a reference ray in the system lying in the plane of symmetry. It is the ray that defines the center of the aperture stop, the center of the field of view and the centers of the pupils. Three special vectors are defined: the normalized aperture vector ($\rho$), which specifies a point in the system aperture and has a foot at the intersection of the OAR and the exit pupil plane of the system, the normalized field vector ($H$), which specifies a point in the system field of view and has a foot at the intersection of the OAR and the image plane, and the unit vector ($i$), which specifies the direction of plane symmetry.

For a surface in the system, the normal to the surface and the OAR make an angle ($I$) in the point of intersection. This angle is measured from the surface normal and is positive if a counterclockwise rotation of the normal is required to reach the OAR. The aberration function can be written as [4]

$$W(H,\rho) = \sum_{k,m,n,p,q} W_{2k+n+p,2m+n+q,n,p,q}(H \cdot H)^k(p \cdot p)^m(i \cdot i)^n, \quad (30)$$

where $W_{2k+n+p,2m+n+q,n,p,q}$ is the coefficient of a particular aberration defined by integers $k,m,n,p,q$. Groups of aberrations are defined by setting the sum of those integers to 0,1,2, etc.

The above equation is a generalization of the classical wave aberration function for an axially symmetric system. When $p$ and $q$ are set to zero, the aberration function for rotationally symmetric systems is recovered.
The terminology of the elements (uniform, linear, etc) is related to the field dependence of the specific aberration. Parameters $\alpha$ and $\beta$ represent the angles between the symmetry unit vector and the field and aperture vectors, respectively. Angle $\varphi$ is the angle between the field and aperture vectors and $\varphi = \alpha - \beta$. The polar coordinate was chosen because it makes the axially symmetric systems appear as a subgroup and seems to be the most appropriate coordinate for system with spherical surfaces. The arrangement of the terms follows the summation $k + m + n + p + q$ because it was the most representative for the aberrations of plane-symmetric systems with spherical surfaces.

The subgroup arrangement allows understanding the new field dependences that arise from misalignments in two-mirror systems, since the break of symmetry caused purely by tilt or decenter turn these systems plane-symmetric.

In plane-symmetric optical systems coma may have two terms: linear coma and uniform coma, respectively:

$$W_{13100}(H \cdot \rho)(\rho \cdot \rho),$$

$$W_{03001}(i \cdot \rho)(\rho \cdot \rho).$$

For an aspheric surface, the total uniform coma coefficient is given by a summation of the spherical base surface and the aspheric “cap” contributions [4]:

$$W_{03001} = W_{03001}^{\text{cap}} + W_{03001}^{\text{sphere}}.$$  

(33)

The spherical contribution to the uniform coma coefficient when a tilt $(I)$ about the surface vertex occurs is given by

$$W_{03001}^{\text{sphere}} = \sum_{i=1}^{i=n} J_{i}^{(i)},$$

(34)

where

$$J_{i} = \frac{1}{2} n \sin(I) A \Delta \left( \frac{u}{n} \right).$$

(35)

The refraction invariant for the paraxial ray, $A$, is given by

$$A = n i = n \left( u + \frac{\chi \cos(I)}{R} \right),$$

(36)

where $R$ is the radius of curvature of the surface - it is positive if the center of curvature lies on the right of the surface - $u$ is the marginal ray slope and $\chi$ is the marginal ray height at a particular surface. The difference operator gives

$$\Delta \left( \frac{u}{n} \right) = \frac{u'}{n'} - \frac{u}{n}.$$  

(37)

Sasian [4] described the sag of an aspheric surface as

$$Z = Z_\text{sphere} + Z_\text{asphere},$$

(38)

where $Z_\text{sphere}$ contribution is given by the sag of a base sphere of radius $R$ and the sag of the aspheric cap, $Z_\text{asphere}$, can be described by

$$Z_\text{asphere} = \alpha (i \cdot p)^2 + \beta (i \cdot p)(p \cdot p) + \gamma (p \cdot p)^2.$$  

(39)
In the equation above, $\alpha$, $\beta$ and $\gamma$ represent three basic surface shapes: cylindrical paraboloid, comatic surface and fourth-order axially symmetric surface, respectively. The composition of these basic surfaces and the spherical surface can approximately describe an aspheric surface.

The aspheric contribution of the tilted surface to uniform coma aberration is given by

$$W_{3000} = \beta \Delta \left[ n \cos(I) \right] x^3.$$  \hspace{1cm} (40)

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