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Parametric analysis of effective material properties of thickness-shear piezoelectric macro-fibre composites

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Introduction

Over the last decade, packaged conformable piezoelectric sensors and actuators have drawn great attention from the research community working on applications such as structural control, structural health monitoring and energy harvesting. For practical and commercial applications, they possess a number of advantages over the standard monolithic piezoelectric patches, such as a less brittle behaviour and more robust electric connection (Skinner, Newnham and Cross, 1978). Based on the concept of Piezoelectric Fibre Composites (PFC) proposed by Bent and Hagood (1997), who used extruded piezoceramic fibres with circular cross-section embedded to an epoxy-based resin matrix and covered by Copper electrodes and protective Kapton and/or Acrylic layers, the so-called Macro-Fibre Composites (MFC) have become very popular since they combine the conformability of epoxy-matrix composites and the electromechanical energy density of piezoceramic materials (Wilkie, Bryant and High, 2000). The MFCs replace the extruded piezoceramic fibres of PFC by machined (diced) rectangular fibres of a piezoceramic material. This innovation led to a cheaper and more reliable manufacturing process and allowed direct contact between fibres and electrodes, solving the main problem of permittivity mismatch of PFCs. The original MFC idea developed at NASA (Wilkie, Bryant and High, 2000) uses Interdigital Electrodes (IDE) to induce longitudinal or 33 mode in the fibres. But, nowadays, it is possible to find MFCs using uniform field (continuous) electrodes that operate with the transverse or 31 mode in the fibres.

Beside the two above longitudinal and transverse piezoelectric responses, the third one is the so-called transverse shear mode, or thickness shear mode, which can be obtained by the application of an electric field that is perpendicular to the remanent polarization direction. This leads to a rotation of the electrical dipoles which induces shear stresses/strains in the material. The transverse shear mode in commercially available piezoceramic patches is normally induced, in the thickness direction, thus perpendicular to the length or width poling direction. Since the mid-90s, the shear mode of monolithic piezoceramic materials has been considered for applications in the design of smart structures (Sun and Zhang, 1995; Benjeddou, Trindade and Ohayon, 1997; Benjeddou, 2007; Trindade and Benjeddou, 2009), including active (Benjeddou, Trindade and Ohayon, 2000; Raja, Prathap and Sinha, 2002; Baillargeon and Vel, 2005), passive (Benjeddou and Ranger, 2006; Trindade and Maio, 2008) and active-passive (Trindade, 2011; Santos and Trindade, 2011) vibration control. From these studies, the shear mode seems to be an interesting alternative for stiffer structures, higher frequencies and non-standard shape deflection patterns.

To take advantage of the high piezoelectric coupling constant $d_{33}$ that most piezoceramic materials have and the fact that it couples directly to transverse shear strains, an alternative MFC design, in which the macro-fibres are oriented perpendicular to the direction of motion, was recently proposed by Raja and Ikeda (2008) to induce the transverse shear mode (15 or 35) in the piezoceramic fibres. It was named as Shear Actuated Fibre Composite (SAFC). The same authors later applied theoretically the proposed design for the vibration control of laminate structures (Raja, Ikeda and Dwarkanathan, 2011). The SAFC basically consists of piezoceramic fibres with poling direction perpendicular to the fibre longitudinal axis, as for the $d_{33}$ MFC, but instead of orienting the fibres along the longitudinal direction of the actuator, the fibres are oriented along the width of the actuator so that poling direction is in the longitudinal direction of the actuator, as shown in Fig. 1. Different from the IDE proposition of Raja and Ikeda (2008), a continuous (Fig. 1a) or woven-type (Fig. 1b) electrode, such as those used in commercial $d_{33}$ MFCs, can be used to induce a uniform electric field in the direction of the actuator thickness (Benjeddou and Al-Ajmi, 2009; Trindade and Benjeddou, 2011). As the fibres are poled in their width direction, that is, the longitudinal direction of the actuator, a shear strain is induced in the fibres transversal planes. The width-poled piezoceramic fibres can be made by dice cutting either thickness-poled or longitudinal- or width-poled piezoceramic plates. However, in the first case, proposed by Raja and Ikeda (2008), the resulting fibres should be rotated 90° around their axes before positioning them in the Epoxy-Copper electrode layer, so that the poling direction of each fibre coincides with the longitudinal direction of the actuator, which can
be difficult in practice. In the second case, several width-poled plates can be aligned in the polymer film frame before dice cutting of the ensemble leading to a probably cheaper process to produce aligned fibres poled in the width direction, which after packaging would be the longitudinal direction of the actuator.

**Figure 1.** Schematic representations of thickness-shear “d15” Macro-Fibre Composite with (a) continuous or homogeneous electrodes (Trindade and Benjeddou, 2011) and (b) discontinuous or heterogeneous electrodes (presently proposed).

One of the main difficulties in the study of MFCs, as an alternative to monolithic piezoceramic patches, is that their behaviour may be much more complex since they are made of several different materials (piezoceramic fibres, epoxy matrix, electrode layers and protective layers). Therefore, it is necessary to understand and model their behaviour to be able to quantify or characterize their effective material properties and, thus, their effectiveness as distributed sensors and actuators. Recently, some research effort has been directed towards the identification and characterization of such transducers, in extension (33 or 31) (Deraemaeker et al., 2007, 2009; Deraemaeker and Nasser, 2010; Biscani et al., 2011) and transverse shear (15) (Benjeddou and Al-Ajmi, 2009) modes. It has been shown that the effective properties depend not only on the piezoceramic material used for the fibres and epoxy material used for the matrix and epoxy-to-piezoceramic volume fraction, but also on the geometrical and material properties of the other layers (Kapton, Acrylic, Electrodes). Some analytical methods from the composite materials literature, such as the Asymptotic Homogenization Method (AHM) (Benjeddou and Al-Ajmi, 2009; Otero et al., 2005) and the Uniform Field Method (UFM) (Bent and Hagood, 1997) can be applied to obtain the effective properties of the composite transducer active core from the properties of its components. Alternatively, one may use techniques based on finite element modelling of the composite transducer active core to identify its effective properties (Berger et al., 2005).

A previous study of a shear actuated d15 MFC made of seven layers (Kapton, Acrylic, Electrode, Piezoceramic Fibre and Epoxy Composite, Electrode, Acrylic, Kapton) using a finite element homogenization method to characterize its effective material properties was presented in Trindade and Benjeddou (2011). It has been shown that the packaging reduces significantly the shear stiffness of the piezoceramic material and, thus, leads to significantly smaller effective electromechanical coupling coefficient k15 and piezoelectric stress constant e15 when compared to the piezoceramic fibre properties. However, it was found that the piezoelectric charge constant d15 is less affected by the softer layers required by the MFC packaging. This work presents, first, an analysis similar to the previous one (Trindade and Benjeddou, 2011) but using another electrode design and, second, a parametric analysis in which the effect of the variations of fibre volume fraction, Epoxy elastic modulus, electrode thickness and active layer thickness on the effective material properties is evaluated.

### Shear Electromechanical Coupling

The shear or d15 response mode requires the poling vector P and main electric field E to be perpendicular to each other. From the modelling point of view, this can be achieved as in Fig. 1, by positioning reference axes such that the poling direction, width of fibre or length of actuator, coincides with direction x3 or z and the fibre thickness, and actuator thickness, coincides with direction x1 or x. These reference system and electric configuration provide easy results interpretation as compared to commercial MFCs and standard piezoelectricity notation.

For a linear orthotropic piezoelectric material poled in x1 direction and electrodes fully covering top (x1) and bottom (x1) surfaces such that a preferential or dominant x1 direction is imposed for the electric field and displacement, so that $E_2 = E_3 = 0$, the constitutive equations can be written in the mixed ε-form type, as

\[
\begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_5 \\
T_6 \\
D_1 \\
D_2 \\
D_3 \\
D_4 \\
D_5 \\
D_6
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} & 0 & 0 & 0 & 0 & 0 \\
\varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} & 0 & 0 & 0 & 0 & 0 \\
\varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \varepsilon_{44} & 0 & 0 & -\varepsilon_{15} & 0 \\
0 & 0 & 0 & 0 & \varepsilon_{55} & 0 & \varepsilon_{35} & 0 \\
0 & 0 & 0 & 0 & 0 & \varepsilon_{66} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_{11} & 0 \\
0 & 0 & 0 & 0 & 0 & \varepsilon_{15} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
S_1 \\
S_2 \\
S_3 \\
S_4 \\
S_5 \\
S_6 \\
E_1 \\
E_2 \\
E_3 \\
E_4 \\
E_5 \\
E_6
\end{bmatrix}
\]

where $T_p$ and $S_q$, with $p, q = 1, \ldots, 6$, denote the six components of mechanical stress and strain vectors in Voigt notation. $D_1$ and $E_1$ denote the electric displacement and field. $\varepsilon_{pq}, \varepsilon_{15}$ and $\varepsilon_{35}$ denote the elastic stiffness (at constant electric fields), piezoelectric and electric permittivity (at constant mechanical strains) constants.

The electromechanical coupling is then between the electric field and displacement in x1 direction, $E_1$ and $D_1$, and $x_1-x_3$ shear stress and strain, $T_5$ and $S_5$. Thus, this is known as ‘15’ operation mode. The electromechanical coupling coefficient can be defined from the value of the coupling or interaction coefficient relative to the principal or diagonal coefficients (Ikeda, 1990) using the standard intensive d-form type of the constitutive equations, such that

\[
k_{15}^2 = \frac{d_{15}^2}{\varepsilon_{35}^T\varepsilon_{11}},
\]

where

\[
d_{15} = \varepsilon_{15}/\varepsilon_{35}, \quad \varepsilon_{35}^T = 1/\varepsilon_{35}^E \quad \text{and} \quad \varepsilon_{11}^T = \varepsilon_{11}^S + d_{15}^2/\varepsilon_{35}^E.
\]

It is thus clear that the material properties $d_{15}$, $\varepsilon_{35}$ and $\varepsilon_{11}^T$ are of major importance for the evaluation of the potential of such material as a candidate for the development of actuators and sensors. Table 1 presents typical values of these constants for a number of commercially available piezoceramic materials.

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Finite Element Homogenization Technique

In this section, the recently proposed (Trindade and Benjeddou, 2011) finite element homogenization technique is applied to evaluate the effective properties of a $d_{15}$ MFC including the electrode and protective layers, according to the schematic representation of Fig. 1(b) that shows the new electrode configuration adopted here. It consists in imposing relative mechanical displacement conditions, including symmetry (or periodicity), on the boundaries of the RVE ($X_1^+, X_1^-, X_2^+, X_2^-, X_3^+, X_3^-$). In addition, electric potential boundary conditions are imposed at the electrodes on surfaces $X_1^+$ and $X_1^-$. Relative potentials are also imposed on the other boundaries as symmetry (or periodicity) conditions. In case of the RVE considered for the $d_{15}$ MFC, shown in Fig. 2, these boundaries are $X_1^+; x_1 = 0, X_1^-; x_1 = h + 2(h_E + h_A + h_K), X_2^+; x_2 = 0, X_2^-; x_2 = w, X_3^+; x_3 = 0, X_3^-; x_3 = L_E + L_E, X_1^{+\prime}; x_1 = h_E + h_A + h_K, X_1^{-\prime}; x_1 = h + h_E + h_A + h_K$.

The protective layers, made of Kapton and Acrylic materials, and electrode layer, made of Copper and Epoxy, are considered isotropic. Notice that the isotropy of the electrode layer depends on its design. Electrode designs such as those used for $d_{33}$ MFC (continuous or woven-type) seem to be more reasonable than those used for the $d_{31}$ MFC (interdigitated), since the voltage should ideally be constant all over the fibre surfaces. An isotropic continuous electrode (Fig. 1(a)) was considered in a previous work (Trindade and Benjeddou, 2011), but, in this work, a discontinuous electrode (Epoxy/Copper/Epoxy) is considered (Fig. 1(b)).

Average strains and electric fields can therefore be imposed to the RVE using displacements and voltage boundary conditions, such that

$$u_i^{+} - u_i^{-} = \bar{S}_{ij}(x_j^{+} - x_j^{-}), \quad i, j = 1, 2, 3,$$

and

$$\phi^{+} - \phi^{-} = -\bar{E}_1(x_1^{+} - x_1^{-}).$$

The resulting average strains, stresses, electric fields and electric displacements in the RVE are defined as

$$\bar{S}_q = \frac{1}{V} \int_V S_q \, dV, \quad \bar{T}_p = \frac{1}{V} \int_V T_p \, dV, \quad \text{with } p, q = 1, \ldots, 6, \quad \text{(6)}$$

where $E_k = \frac{1}{V} \int_V E_k \, dV, \quad D_i = \frac{1}{V} \int_V D_i \, dV, \quad \text{with } i, k = 1, 2, 3. \quad \text{(7)}$

These integrals are approximated in Ansys(R) by a sum over averaged element values multiplied by the respective element volume divided by total volume of the RVE, such that

$$\bar{S}_q = \frac{\sum_{e=1}^N S_{e}^{(e)} V(e)}{\sum_{e=1}^N V(e)}, \quad \bar{T}_p = \frac{\sum_{e=1}^N T_{p}^{(e)} V(e)}{\sum_{e=1}^N V(e)}, \quad \text{with } p, q = 1, \ldots, 6. \quad \text{(8)}$$

These average stresses, strains, electric fields and electric displacements evaluated at element $e$. $N$ is the total number of finite elements used to discretize the RVE.

For the evaluation of the elastic constants related to normal strains and stresses $c_{pq}^{E}, \quad p, q = 1, 2, 3$, three local problems are analysed, for which a normal strain $\bar{S}_q = (q = 1, 2, 3)$ is applied by imposing relative normal displacements $u_q - u_q$. In order to obtain $\bar{S}_j = 0$ if $j \neq q$, the normal displacements $u_j$ for each pair of nodes at opposing surfaces $X_j^+$ and $X_j^-$ are set to be equal (symmetrically coupled degrees of freedom). Then, considering the constitutive equations in (1), the effective elastic constants are evaluated using the following expressions

$$c_{pq}^{E} = \bar{T}_{p}/\bar{S}_q, \quad p, q = 1, 2, 3, \quad \text{(10)}$$

where the average stresses $\bar{T}_{p}$ and strains $\bar{S}_q$ are evaluated using (8).

For the evaluation of the elastic constants related to shear strains and stresses $c_{pq}^{E}, \quad p = 4, 5, 6$, other three local problems are analysed for which shear strains $\bar{S}_{pq} = (p+q)/2, \quad \bar{S}_{12} = \bar{S}_{21}/2$ and $\bar{S}_{13} = \bar{S}_{31}/2$ are applied to approximate a pure shear stress state in the planes $x_2 \times x_3, x_1 \times x_3$ and $x_1 \times x_2$ by imposing relative shear displacements $u_q - u_q$ at surfaces $X_q^+ / X_q^-$ and $u_q - u_q$ at surfaces $X_q^+ / X_q^-$. Moreover, the normal displacements at the boundary surfaces parallel to each shear plane of interest are symmetrically coupled to ensure zero strain perpendicular to the shear plane. Therefore, the effective elastic constants are evaluated in terms of the average shear stresses $\bar{T}_{p}$ and strains $\bar{S}_p$ using

$$c_{pq}^{T} = \bar{T}_{p}/\bar{S}_p, \quad p = 4, 5, 6. \quad \text{(11)}$$

<table>
<thead>
<tr>
<th>Material</th>
<th>$c^{E}_{11}$(GPa)</th>
<th>$c^{E}_{12}$(GPa)</th>
<th>$c^{E}_{13}$(GPa)</th>
<th>$c^{E}_{15}$(C/m$^2$)</th>
<th>$c^{E}_{31}$(%)</th>
<th>$c^{E}_{33}$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>APC-840</td>
<td>26.9</td>
<td>1427</td>
<td>13.6</td>
<td>49.0</td>
<td>0.70</td>
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<tr>
<td>APC-850</td>
<td>21.8</td>
<td>1851</td>
<td>13.2</td>
<td>46.2</td>
<td>0.68</td>
<td></td>
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<tr>
<td>APC-855</td>
<td>22.4</td>
<td>3012</td>
<td>15.0</td>
<td>43.6</td>
<td>0.66</td>
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<tr>
<td>EC-76</td>
<td>21.9</td>
<td>2853</td>
<td>16.0</td>
<td>46.2</td>
<td>0.68</td>
<td></td>
</tr>
<tr>
<td>EC-64</td>
<td>25.7</td>
<td>1436</td>
<td>13.0</td>
<td>51.8</td>
<td>0.72</td>
<td></td>
</tr>
<tr>
<td>PIC-255</td>
<td>21.0</td>
<td>1650</td>
<td>11.6</td>
<td>43.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PZT-5H</td>
<td>23.0</td>
<td>3130</td>
<td>17.0</td>
<td>45.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PZT-5A</td>
<td>21.1</td>
<td>1730</td>
<td>12.3</td>
<td>49.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PZT-7A</td>
<td>29.4</td>
<td>930</td>
<td>10.6</td>
<td>46.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sonox-P502</td>
<td>30.1</td>
<td>1950</td>
<td>16.9</td>
<td>54.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sonox-P504</td>
<td>24.8</td>
<td>1920</td>
<td>13.2</td>
<td>43.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sonox-P508</td>
<td>29.8</td>
<td>1700</td>
<td>16.4</td>
<td>50.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For both the previous cases, equations (10) and (11), to ensure a short circuit electric boundary condition, the voltage degrees of freedom in the electrodes at surfaces \( X_1^+ \) and \( X_1^- \) are set to zero. Table 2 indicates symmetric boundary conditions for the electric potential along directions \( x_2 \) and \( x_3 \) for the corresponding \( X_2^+/X_2^- \) and \( X_3^+/X_3^- \) RVE surfaces.

The piezoelectric constant \( \varepsilon_{15} \) can be also obtained from the local problem used to evaluate \( c_{55} \), which is where a pure shear stress state in the plane \( x_1-x_3 \) is approximated. From the constitutive equations (1) and (2), \( \varepsilon_{15} \) can be obtained by evaluating the average electric displacement \( D_1 \) such that

\[
\varepsilon_{15} = \frac{D_1}{S_S}.
\]

(12)

where the average shear strain \( S_S \) and electric displacement \( D_1 \) are evaluated using the approximation of (8) and (9).

For the evaluation of the piezoelectric and dielectric constants of the \( d \)-form, \( d_{15} \) and \( e_{11}^T \), another local problem is set up for which an electric voltage \( \phi_{X_1^+} = h_F \) (in value) is applied to the electrode at surface \( X_1^+ \) (with \( x_3 = h_F + h_A + h_G \)) while the voltage at the opposite electrode (at surface \( X_1^- \), \( x_3 = h_G + h_A + h_F \)) is set to zero so that a unitary electric field in the \( x_1 \) direction is generated. To approximate the condition of zero stresses in the RVE, no restriction is made to the displacements in the RVE except for the origin where all displacements are set to zero to prevent rigid body displacements.

Then, considering zero shear stress \( T_2 \) in the \( d \)-form of the constitutive equations, the piezoelectric and dielectric constants, \( d_{15} \) and \( e_{11}^T \), can be evaluated from

\[
d_{15} = \frac{S_S}{E_1} \quad \text{and} \quad e_{11}^T = \frac{D_1}{E_1}.
\]

(13)

Table 2 summarizes the local problems used for the characterization of the relevant material properties of the \( d_{15} \) MFC RVE. \( u_j^- \) and \( u_j^+ \) are the displacements in direction \( j \) and \( \phi_j^- \) and \( \phi_j^+ \) are the electric potentials at \( i \)-th node of opposite surfaces. \( q \) is an arbitrary non-null value.

**d_{15} MFC Effective Material Properties Evaluation**

Figure 3 shows the finite element mesh used in Ansys? for the 7-layered \( d_{15} \) MFC RVE using a FVF of 0.86 for the PZT+Epoxy Active Layer. The 3D 20-Node coupled-field solid finite element SOLID226 was used to mesh all volumes. It has the three displacement translations along the Cartesian coordinate system and the electric potential as nodal degrees of freedom. 5525 finite elements were used considering 25 divisions in \( x_1 \) direction (13 divisions for the PZT layer and 2 divisions for each electrode and protective layer), 13 divisions in \( x_2 \) direction and 17 divisions in \( x_3 \) direction (11 divisions for the PZT layer and 3 divisions for each Epoxy layer). According to Fig. 2, the dimensions considered for this RVE are \( h_P = 1 \) mm, \( h_F = 25.4 \) mm, \( h_A = 12.7 \) \( \mu \)m and \( h_E = 17.8 \) \( \mu \)m, \( L_P = 0.860 \) mm, \( L_E = 0.140 \) mm and \( w = 1 \) mm (Raja and Ikeda, 2008). The material properties for the piezoceramic material Sonox P502, taken from Deramaeker et al. (2009), are: \( s_{11}^E = 13.8 \) pm/N, \( s_{22}^E = 20.7 \) pm/N, \( s_{12}^E = -6.29 \) pm/N, \( s_{13}^E = -6.23 \) pm/N, \( s_{44}^E = 33.2 \) pm/N, \( s_{15}^E = 52.3 \) pm/N, \( s_{33}^E = 440 \) pCN, \( d_{15} \) = 560 pCN, \( e_{11}^T = e_{22}^T = 1950 \) GPa, \( e_{33}^T = 1850 \) GPa. The material properties for the isotropic Epoxy are: \( Y = 2.9 \) GPa, \( \nu = 0.3 \) and \( e = 4.25 \mu \)m. The materials properties for the protective layers, taken from Raja and Ikeda (2008), are: Kapton – \( Y = 2.5 \) GPa, \( \nu = 0.34 \) and \( e = 3.4 \mu \)m and Acrylic – \( Y = 2.7 \) GPa, \( \nu = 0.35 \) and \( e = 3.4 \mu \)m. The material properties for the Copper electrode are: \( Y = 117 \) GPa, \( \nu = 0.35 \) and \( e = 5 \mu \)m.

![Figure 2. Representative volume element (RVE) for a \( d_{15} \) MFC.](image-url)

Table 3 shows the effective properties, transformed to Voigt notation, of the \( d_{15} \) MFC, active layer only (columns 1 and 3) and full 7-layer composite (columns 2 and 4), using FVF of 0.86 and 0.95 obtained by the finite element local problems. The effective electromechanical coupling coefficient of the RVE \( k_{15} \) is evaluated from the effective elastic, piezoelectric and dielectric properties, \( c_{55}^E \), \( d_{15} \) and \( e_{11}^T \), using (2). The results for the active layer only were validated in a previous work (Trindade and Benjeddou, 2011) through comparison with those obtained by analytical homogenization methods UFM (without the unidirectional electric field, UDEF, approximation) and AHM (with the UDEF
The shear modulus $G$ instance a decrease on the thickness of these layers should increase layers should also play a major role in this shear modulus, for FVF 0.95) due to the protective layers which are softer than the PZT. modulus diminished due to the relatively soft protective layers. On the other hand, the Young modulus substantially by 10% and 123%, respectively, for the active layer. This properties of major interest to the $d_{33}$, increase, leads to an overall increase in the elastic properties were evaluated using mixture laws, whereas, in the present case, the electrode layer is heterogeneous (Epoxy/Copper/Epoxy) and the Copper portion covers only the piezoceramic fibre, as shown in Figs. 1(b) and 2. The present results are almost exactly the same as in Trindade and Benjeddou (2011).

![Finite element mesh for the 7-layered $d_{33}$ MFC RVE with FVF = 0.86.](Image)

The results show that the increase in the FVF from 0.86 to 0.95, meaning a 10% increase, leads to an overall increase in the elastic (stiffness) constants. More importantly, the effective piezoelectric properties of major interest to the $d_{33}$ MFC, $d_{15}$ or $e_{15}$, increase substantially by 10% and 123%, respectively, for the active layer. This yields an increase in the effective squared electromechanical coupling coefficient $k_{15}^2$ by 123%, from 10% to 23%, for the active layer.

Table 3 also shows that, while the active layer is almost transversally isotropic, the 7-layered composite is definitely orthotropic. In particular, the Young modulus $Y_1$ is significantly diminished due to the relatively soft protective layers. On the other hand, the Young modulus $Y_3$ is little decreased mainly due to the electrode that is stiffer than the PZT core and thus promotes a stiffer connection between the $X_3$ surface and the PZT core. The shear modulus $G_{13}$ is diminished by 27% (for FVF 0.86) and 46% (for FVF 0.95) due to the protective layers which are softer than the PZT. Notice, however, that the thickness of the protective and electrode layers should also play a major role in this shear modulus, for instance a decrease on the thickness of these layers should increase the shear modulus $G_{13}$, but will also perform poorly as protective layers. The piezoelectric constant $d_{15}$ is decreased a little while $e_{15}$ is decreased by 30%, from 3.13 to 2.06 C/m$^2$. The dielectric constant $\epsilon_{11}$ is not modified due to the protective and electrode layers, since it is assumed that the electric contact between electrode and PZT occurs at the PZT-electrode interface. This would not be the case if there should be an epoxy layer between Copper electrode and PZT. These effective properties yield an overall smaller electromechanical coupling coefficient $k_{15}$, such that the efficiency in energy conversion $k_{15}^2$ decreases by 20%, from 10% to 8%. Notice that this is much less than the $k_{15}^2$ of the PZT (Sonox P502) alone of 54.8%.

### Parametric Analysis

In the previous section, it was noticed that, besides the fibre volume fraction, the packaging design may affect significantly the performance of the MFC transducer. In particular, the softer materials that provide the desired flexibility for the transducer also decrease its performance. Therefore, it is of great interest to analyse the effect of the variation of material and geometrical properties of the MFC on its effective material properties.

In particular, it was observed that the Epoxy material is the main responsible for the decrease of MFC stiffness and, thus, actuation authority represented by the piezoelectric stress constant $e_{15}$. The MFC effective stiffness can be controlled through either the fibre volume fraction or the Epoxy material stiffness constants, so that the stiffness in energy conversion $k_{15}^2$ increases by 20%, from 10% to 8%. Notice that this is much less than the $k_{15}^2$ of the PZT (Sonox P502) alone of 54.8%.

![Finite element mesh for the 7-layered $d_{33}$ MFC RVE with FVF = 0.86.](Image)

**Table 3. Effective short-circuit material properties of $d_{33}$ MFC and its active layer for two fibre volume fractions.**

<table>
<thead>
<tr>
<th>MFC active layer</th>
<th>7-layers MFC</th>
<th>MFC active layer</th>
<th>7-layers MFC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_1 = 50.09$ GPa</td>
<td>$Y_1 = 25.43$ GPa</td>
<td>$Y_1 = 54.81$ GPa</td>
<td>$Y_1 = 28.72$ GPa</td>
</tr>
<tr>
<td>$Y_2 = 50.09$ GPa</td>
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Parametric Analysis of Effective Material Properties of Thickness-Shear Piezoelectric Macro-Fibre Composites

volume fraction varies from 68% to 95% with 3% increments. The Epoxy elastic modulus is decreased/increased by 10% and 20% from the standard value used in the previous section ($E_E = 2.9$ GPa). The Copper electrode thickness is also decreased/increased by 10% and 20% from the value of the previous section ($h_C = 17.8$ μm). As for the active layer thickness, it varies from 0.25 mm to 1.0 mm with 0.25 mm increments. Notice that, for each parameter set, the seven local problems presented previously (Table 2) are solved and the corresponding effective material properties are evaluated.

In a first analysis, the fibre volume fraction varies in the range from 68% to 95%. Notice that the standard piezoelectric MFC are manufactured using a 86% FVF (Deraemaeker et al., 2007). As expected, all elastic stiffness coefficients increase for higher FVF, since the piezoceramic is much stiffer than the Epoxy (Fig. 4). A more detailed analysis shows, however, that whereas constants $Y_1$, $Y_2$ and $G_{12}$ increase linearly with the FVF, the other elastic stiffness constants $Y_3$, $G_{23}$ and $G_{13}$ present a non-linear augmentation, as shown in Fig. 4. On the other hand, the Poisson ratios $\nu_{13}$ and $\nu_{12}$ decrease with increasing FVF and $\nu_{23}$ increases moderately. As shown in Fig. 5, the piezoelectric and dielectric material properties increase substantially for higher FVF, since those are only due to the piezoelectric part of the active layer. It can be observed from Fig. 5 that $\varepsilon_{11}^e$ and $d_{15}$ (dielectric and piezoelectric) constants increase linearly with FVF, while piezoelectric constant $e_{15}$ and squared electromechanical coupling coefficient $k_{15}^2$ increase rates augment with FVF, indicating that maximization of FVF is more important for applications using the $d_{15}$ MFC as actuators or energy converters.

In a second analysis, the effect of the Epoxy elastic modulus on the effective material properties is analyzed. Since the major part of elastic strains is concentrated in the Epoxy material, it is expected that stiffer Epoxy materials could improve both overall stiffness and electromechanical coupling. Indeed, as shown in Fig. 6(a), an increase of 20% on the Epoxy elastic modulus, relative to the standard value $E_E = 2.9$ GPa, leads to an increase of almost 15% on $d_{15}$ MFC elastic constant $Y_3$. On the other hand, effective elastic constants $Y_1$ and $Y_2$ are not significantly altered by the Epoxy elastic modulus. As for the shear elastic constants, $G_{23}$ and $G_{13}$ increase by around 15% and 10%, respectively, when Epoxy elastic modulus is increased by 20% while constant $G_{12}$ remains nearly unchanged (Fig. 6(b)). The Poisson ratios vary by less than 5% as shown in Fig. 6(c).

On the other hand, the dielectric constant $\varepsilon_{11}^T$ of the $d_{15}$ MFC is not affected by the variation of Epoxy elastic modulus (Fig. 7(a)). The same happens to piezoelectric constant $d_{15}$ (Fig. 7(b)). However, the piezoelectric constant $e_{15}$, which depends on the effective shear stiffness of the composite, increases approximately by 10% when the Epoxy elastic modulus is increased by 20%, as shown in Fig. 7(c). A very similar dependence is observed for the squared electromechanical coupling coefficient $k_{15}^2$, shown in Fig. 7(d). This indicates that stiffer Epoxy materials should be more interesting for actuators and energy converters applications. Notice, however, that this would also lead to a less flexible (conformable) transducer.

From previous analyses, it seemed that electrode thickness could affect the effective material properties of the $d_{15}$ MFC, since this layer responds to the major part of the covering layer (electrode and Kapton/Acrylic protective layers) stiffness. Therefore, this property was also varied in the range ±20%. Since a Copper electrode is commonly used, it did not seem appropriate to vary its elastic modulus. For the same reason, the Kapton and Acrylic protective layers material properties were also kept unchanged. Figure 8 shows that an increase of the electrode thickness yields an overall augmentation of the $d_{15}$ MFC stiffness constants, although this augmentation is smaller than 1%. Elastic constants $Y_1, Y_2$ and $Y_{12}$ are the most affected by the electrode thickness variation. Poisson ratios are not significantly affected. As for the variation of the previous parameter, dielectric constant $\varepsilon_{11}^T$ and piezoelectric constant $d_{15}$ are not affected by the variation of electrode thickness. On the other hand, the piezoelectric constant $e_{15}$ decreases a little (0.4%), while the squared electromechanical coupling coefficient $k_{15}^2$ increases a little (0.2%), for an increase of 20% in the electrode thickness (Fig. 9).

In the last parametric analysis, the active layer thickness varies in the range [0.25-1.00] mm while maintaining the other parameters unchanged. Notice that this parameter could also affect the coupling with the external (host) structure since it modifies the transducer geometry and, thus, its effective performance. Nevertheless, the reduction of active layer thickness also leads to a decrease of the active part in relation to the MFC packaging and, thus, it is worthwhile analysing the effect of this variation on the MFC effective properties.

As shown in Fig. 10, both Young and shear moduli augment for increasing active layer thickness. Constants $Y_1, G_{12}$ and $\nu_{12}$ are the most affected by this variation. The dielectric constant $\varepsilon_{11}^T$...
and piezoelectric charge constant $d_{15}$ are not much affected by the variation of the active layer thickness, increasing around 2% for a decrease on the active layer thickness from 1 mm down to 0.25 mm (Figs. 11(a) and 11(b)). On the other hand, both piezoelectric stress constant $e_{15}$ and squared electromechanical coupling coefficient $k_{15}^2$ increase substantially for increasing active layer thickness, around 100% for $e_{15}$ and 50% for $k_{15}^2$ (Figs. 11(c) and 11(d)).

**Concluding Remarks**

This work presented a parametric analysis of the effective material properties of a $d_{15}$ thickness-shear piezoelectric Macro-
Fibre Composite (MFC) made of seven layers (Kapton, Acrylic, Electrode, Piezoceramic Fibre and Epoxy Composite, Electrode, Acrylic, Kapton). It confirms that the MFC packaging reduces significantly the shear stiffness of the piezoceramic material and, thus, leading to significantly smaller effective electromechanical coupling coefficient $k_{15}$ and piezoelectric stress constant $\epsilon_{15}$ when compared to the piezoceramic fibre properties. The effect of the variations of fibre volume fraction, Epoxy elastic modulus, electrode thickness and active layer thickness on the MFC effective material properties was evaluated. Results indicated that an effective $d_{15}$ MFC should use relatively thick fibres having relatively high shear modulus and relatively stiff epoxy filler. On the other hand, it was found that the electrode thickness does not affect significantly the MFC performance.

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References


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