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PD controller synthesis from open-loop response measurements of rotating system

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Abstract: In this work, a method of computing PD stabilising gains for rotating systems is presented based on the D-decomposition technique, which requires the sole knowledge of frequency response functions. By applying this method to a rotating system with electromagnetic actuators, it is demonstrated that the stability boundary locus in the plane of feedback gains can be easily plotted, and the most suitable gains can be found to minimise the resonant peak of the system. Experimental results for a Laval rotor show the feasibility of not only controlling lateral shaft vibration and assuring stability, but also helps in predicting the final vibration level achieved by the closed-loop system. These results are obtained based solely on the input–output response information of the system as a whole.

1 Introduction

Rotating machines are vital elements in the industry and, for this reason, they must present not only high performance, but also high availability to avoid interruptions in the production flow. One way of increasing the efficiency of rotating machines is the attenuation/control of the vibration levels, particularly the vibration that occurs in the shaft. Unbalance, misalignment or external forces owing to operational conditions are the most common contributing factors for high-lateral vibration in rotors, and they can reduce the performance and cause energy loss, fatigue or even failure [1, 2]. However hard it is to eliminate vibration, controlling vibration levels within acceptable margins is essential for the safe and reliable operation of rotating machines.

In this context, actuators and sensors have been incorporated into rotating machines, and control systems have been developed [3–6]. In the literature, one can find different control techniques applied to vibration control of rotating system, most of them based on traditional control strategies: proportional-integral-derivative control (PID), optimum and robust control and adaptive control. A thorough review of control system design for rotating system is presented in [7]. In the experimental implementation of such control systems, it is usually necessary to have a mathematical model of the system to design the controller [8–10]. In these cases, the successful control of vibration depends on the quality of the adopted model, including the model of sensors and actuators. Considering that imprecise models can jeopardize the performance of the controller, a model-free design of controllers began to be investigated.

One of the first works on control design without system modelling refers to linear controllers [11]. In this case, the linear controller is similar to a lead/lag compensator, and the closed-loop stabilising gains of the controller are determined from the frequency response functions (FRFs) of the open-loop system. Later, the theory for designing PID controllers was presented [12], whose equations for finding the stabilising gains were formally deduced in [13]. By knowing the gains that stabilise the closed-loop system, one can find sets of gains that optimise any performance criteria, thus achieving closed-loop stability and robustness [14, 15].

The cases cited above, adopt the D-decomposition technique to find the regions of stabilising gains. The first idea about the D-decomposition are attributed to Vishnegradsky [16], who graphically represented the stability condition of a characteristic polynomial of the form \( \pi(s, K_1, K_2) \) in terms of its parameters \( K_1 \) and \( K_2 \). According to [17], the same ideas were explored by Frazer and Duncan [18] and Sokolov [19], but it was Neimark [20] who developed the algorithm and coined the name D-decomposition. The technique consists of deriving three conditions, which allow one to decompose the parameter space into regions with a fixed number of stable and unstable roots (root invariant regions).

Nowadays, the D-decomposition technique is usually used together with mathematical models to find the gains of linear and \( H_\infty \) controllers [21–25]. However, few works in the literature apply the D-decomposition technique to find the stabilising gains from measured FRFs. In such cases, the actuators can be used as exciters and open-loop frequency response is obtained for the global system composed of the actuator system+plant+sensor system. Hence, all dynamic information is embedded in the global FRFs, and the controller can be designed with no further data [26, 27].

The application of such methodology to rotating systems is a novelty and it is still open for investigations. In this
work, the D-decomposition technique is used to find the stabilising gains of a proportional-derivative control (PD) controller, whose objective is controlling lateral vibration of a rotating system with flexible shaft. The methodology is implemented experimentally and the regions of stabilising gains are obtained from the global frequency response of the open-loop system (actuators+rotor+sensors), where the actuators act as exciters. A Laval rotor configuration is studied, and the performance of the controller with the adopted gains is compared with the expected results from theory. Results show the feasibility of not only controlling lateral shaft vibration and assuring stability, but also helps in predicting the final vibration level achieved by the closed-loop system within acceptable error margins. The obtained results are based solely on the input–output response information of the open-loop system as a whole, from experimental data.

The idea behind this work is the tuning of the PD controller with no previous information of the rotating system. This is especially useful in rotating systems supported by active bearings, where uncertainty plays an important role in system dynamics. In this case, the control system (actuators and sensors) shall identify the system characteristics (dynamics) and decide which gains to use in the PD controller. Later, the gains remain constant, but the procedure could be reused to update the gains after some time interval. Alternatively, the procedure could be used to find the first gains to be adopted in the PD controller, and later, the gains could be updated by an adaptive controller according to the external disturbances. Such a procedure is suitable for systems that can start running the rotor without necessarily turning on the controller, for example, active squeeze film dampers, bearings with active lubrication, bearings with piezo stack actuators, active hydrostatic bearings and any other kind of active hybrid bearing.

2 D-decomposition technique

The basic idea of the D-decomposition approach consists of dividing the controller parameter space into root invariant regions, that is, regions with a fixed number of stable and unstable roots [17]. This division is obtained calculating boundaries, which map the imaginary axis in the complex plane into curves in the controller parameter space. As each region delimited by these boundaries corresponds to a root invariant region, if a set of gains in a certain region leads the system to stability, all gains inside this region will do so as well.

Consider \( \pi(j\omega, \lambda) \) as the characteristic polynomial of a closed-loop system of degree \( n \) written in the frequency domain and with real coefficients \( \alpha_i(\lambda) \), where \( \lambda \in \mathbb{R}^n \) is an uncertain parameter (e.g. controller gains) and \( \omega \in [0, +\infty) \)

\[
\pi(j\omega, \lambda) = \alpha_n(\lambda)(j\omega)^n + \alpha_{n-1}(\lambda)(j\omega)^{n-1} + \cdots + \alpha_0(\lambda)
\]  

(1)

The boundaries of each region are defined by

\[
\pi(0, \lambda) = 0
\]

(2)

\[
\pi(j\omega_0, \lambda) = 0
\]

(3)

\[
\alpha_0(\lambda) = 0
\]

(4)

Equation (2) is known as the real root boundary (RRB) and provides the values of the parameter \( \lambda \) for which the real roots are in the origin. Equivalently, it can be written as

\[
\alpha_0(\lambda) = 0
\]

(5)

Equation (3) is known as the complex root boundary (CRB) and provides the values of the parameter \( \lambda \) for which the complex roots are on the imaginary axis. Equation (4) is known as the infinite root boundary (IRB) and provides the values of the parameter \( \lambda \) for which the roots pass through infinity, that is, this condition maps the other way in which the real roots can cross the imaginary axis (through a reduction in the level of the characteristic polynomial which, consequently, implies change in the number of the poles).

The three previous conditions allow one to decompose the controller parameter space in regions with a fixed number of stable and unstable roots (root invariant regions) [15, 17]. However, it is noteworthy that there is no guarantee of the existence of a region and this set may be empty, or all regions can result in instability [28].

3 PD controller root invariant regions

Consider the feedback configuration with a linear time invariant system and a first-order controller according to Fig. 1. The system and the controller are defined, respectively, as

\[
P(j\omega) = P_r(\omega) + j P_i(\omega) \quad \text{and} \quad C(j\omega) = j\omega G_D + G_P
\]

(6)

where \( G_D \) and \( G_P \) are the proportional and derivative gains of the PD controller, and \( P(j\omega) \) is the frequency response of the system, which is assumed to be controllable. The system’s closed-loop response function, that is, the response function from the input \( F(j\omega) \) to the output \( X(j\omega) \) in the frequency domain, is given by

\[
H(j\omega) = \frac{P(j\omega)}{1 + (j\omega G_D + G_P) P(j\omega)}
\]

(7)

and the characteristic polynomial of the closed-loop system is

\[
\pi(j\omega) = 1 + (j\omega G_D + G_P) P(j\omega) = \pi_r(j\omega) + j \pi_i(j\omega)
\]

(8)

where subscripts \( r \) and \( i \) refer to the real and imaginary parts, respectively, and

\[
\pi_r(j\omega) = 1 + G_P P_r(j\omega) - G_D P_i(j\omega)
\]

\[
\pi_i(j\omega) = G_P P_i(j\omega) + G_D P_r(j\omega)
\]

(9)

\[
\psi(\omega) = \frac{1}{1 + (j\omega G_D + G_P) P(j\omega)}
\]

(10)

\[
\frac{\psi(j\omega)}{\psi(0)} = \frac{1 + (j\omega G_D + G_P) P(j\omega)}{1 + P(j\omega)}
\]

(11)

\[
\omega = \frac{1}{\pi(j\omega)}
\]

(12)

\[
\omega_{\text{crit}} = \frac{1}{\pi_r(j\omega)}
\]

(13)

The system’s closed-loop frequency response function, that is, the response function from the input \( F(j\omega) \) to the output \( X(j\omega) \) in the frequency domain, is

\[
\psi(j\omega) = \frac{1}{1 + (j\omega G_D + G_P) P(j\omega)}
\]

(14)

\[
\frac{\psi(j\omega)}{\psi(0)} = \frac{1 + (j\omega G_D + G_P) P(j\omega)}{1 + P(j\omega)}
\]

(15)

\[
\omega = \frac{1}{\pi(j\omega)}
\]

(16)

\[
\omega_{\text{crit}} = \frac{1}{\pi_r(j\omega)}
\]

(17)
3.1 Real root boundary

Considering the closed-loop characteristic polynomial of the system \((8)\) and the RRB defined by \((2)\) and \((5)\), one has

\[
\pi (\omega = 0, G_D, G_D) = 1 + G_P P(0) = 0
\]

If \(P(0) \neq 0\), then

\[
G_P = -\frac{1}{P(0)}
\]

3.2 Infinity root boundary

Assuming that the denominator of the plant’s transfer function has order \(n\) and the numerator has order \(n - 1\), and taking the IRB defined by \((4)\), one can write the closed-loop characteristic polynomial \((8)\) as

\[
d_n + G_D n_{n-1} = 0
\]

where \(d_n\) and \(n_n\) are the \(n\)th order coefficients of the denominator and the numerator, respectively. Hence, if \(P(\infty) \neq 0\):

\[
G_D = -\frac{d_n}{n_{n-1}} = -\frac{1}{P(\infty)}
\]

In the case \(P(j\omega)\) is obtained experimentally, the IRB is seldom applicable because either information at infinity is not available or system response tends to zero \((G_D \to \infty)\).

3.3 Complex root boundary

According to the CRB defined by \((3), (9)\) can be written as

\[
\begin{bmatrix}
P_r(\omega) -\omega P_r(\omega) \\
P_i(\omega) -\omega P_i(\omega)
\end{bmatrix}
\begin{bmatrix}
G_P \\
G_D
\end{bmatrix}
= \begin{bmatrix}
-1 \\
0
\end{bmatrix}
\]

whose solution is

\[
\begin{align*}
G_P &= \frac{-P_r(\omega)}{|P_r(j\omega)|^2} \\
G_D &= \frac{-P_i(\omega)}{|\omega P(j\omega)|^2}
\end{align*}
\]

Hence, given the frequency response of the system to be controlled \(P(j\omega)\), the controller parameter space \((G_D \times G_P)\) can be partitioned into root invariant regions whose boundaries are defined by \((11), (13)\) and \((15)\). The only information required for the analysis is the frequency response of the open-loop system. To know which partitions correspond to \(D(n, 0)\) regions, that is, regions with \(n\) stable and zero unstable poles, it is necessary to select a point from each region and test the stability of the corresponding closed-loop system.

4 Results for a single disc rotating system

The proposed method is experimentally applied to the single disc rotating system shown in Fig. 2. The test rig is composed of a steel shaft, supported by self-aligning ball bearings, with a concentrated mass in the mid span (steel disc), thus resulting in a Laval rotor configuration (no gyroscopic effect expected). The shaft is driven by an electric motor whose speed is controlled by a frequency inverter. Electromagnetic actuators are located near the disc, acting in horizontal direction (perpendicular to the view shown in Fig. 2). The response of the shaft is measured by proximity probes also mounted near the disc and in the horizontal and vertical directions.

To obtain the open-loop frequency response of the system, the actuating system+shaft dynamics+sensor system will be considered as a whole, whose response is \(P(j\omega)\) (Fig. 1). In this case, all dynamics of the actuating system (drives and actuators), rotating system and sensor system will be embedded in the measured response \(P(j\omega)\) (Fig. 3).

The global frequency response of the open-loop system is measured by sending a chirp signal (in V) to the actuators’ drive and measuring the response (in mm) with the proximity sensors (Fig. 3). The adopted chirp signal ranged from 1 to 40 Hz, going back and forth in 10 s, with an amplitude of 2 V. A NI PCI-6229 acquisition board is used for signal acquisition, and control is managed by a program through MatLab real-time workshop (RTW). Fig. 4 shows the FRFs in the horizontal direction, obtained experimentally for different rotating speeds (waterfall diagram of \(H_i\) estimators [29]). One can clearly see in Fig. 4 the resonance frequency of the shaft near 22 Hz and the synchronous component at the rotating frequency.

By applying the root boundaries defined in \((11)\) and \((15)\) to the global response measured at zero-rotating speed (0 rpm), one obtains the \(G_D \times G_P\) plane shown in Fig. 5. The RRB represents a straight line in the \(G_D \times G_P\) plane, whereas the CRB results in a curve whose coordinates are frequency dependent. The IRB depends on the response of the global system when \(\omega \to \infty\) \((13)\). As \(\omega \to \infty\), the output of the actuator system \((A(j\omega))\) tends to zero (characteristics of inductance systems). However, even for zero input, the response of the rotating system \((R(j\omega))\) at \(\omega \to \infty\) tends to the unbalance eccentricity (self-centring of the shaft). However, the sensors are inductive and work at a base-measuring frequency. Hence, for \(\omega \to \infty\), the sensor will not be able to measure the rotating system response, and the output of \(U(j\omega)\) at \(\omega \to \infty\) will be zero. As a result, the response of the global system at \(\omega \to \infty\) is zero and the IRB will tend to infinity.

Hence, one can see in Fig. 5 that the \(G_D \times G_P\) plane is divided into regions whose boundaries were defined by applying \((11)\) and \((15)\). According to the D-decomposition technique, these regions in Fig. 5 will result in an invariant number of stable and unstable roots of the closed-loop system. A set of gains on the RRB implies that the real poles are on the origin, whereas a set of gains on the CRB implies that the complex roots are on the imaginary axes. The question is, which regions have gains that result in stable roots only.
Fig. 3 Experimental set-up for feedback control system and global system response

Fig. 4 Waterfall diagram of the single disc rotating system (experimental global system response)

Fig. 5 Controller parameter space partitioned into root invariant regions (single disc system at 0 rpm)

4.1 Stability analysis

The root invariant regions of the system for three different rotating speeds is shown in Fig. 6. As the shaft starts whirling, the root invariant regions change, but remain almost unchanged for both subcritical (590 rpm) and supercritical (1770 rpm) rotating speeds. The spikes in vertical direction in the CRB refer to the synchronous component in the frequency response of the system.

Taking sets of gains in different regions of the $G_D \times G_P$ plane (points A–E in Fig. 6), one can test the stability of the system and find the region, or regions, that presents stable roots only. The chosen sets of gains are listed in Table 1, and the Nyquist stability criterion is adopted. In the present case, the open-loop system (flexible rotor) is inherently stable, therefore the closed-loop system will be unstable if, and only if, the Nyquist plot encircles the point $(-1, 0)$. According to the Nyquist stability criterion, gain set A leads to instability, whatever rotating speed is adopted (clockwise encirclement of $-1 + j0$). Gain set B leads to stability in the case of null-rotating speed, but it still leads to instability for non-zero rotating speeds. Gain set C leads to stability, irrespective of the rotating speed adopted (no encirclement of $-1 + j0$ and no poles in the right-half plane). Gain set D leads to stability in the case of non-zero rotating speeds, but it leads to instability in the case of null rotating speed. Finally, gain set E leads to instability, irrespective of the rotating speed adopted (clockwise encirclement of $-1 + j0$). Hence, it is clear that the region of gains below the RRB and within the CRB leads the closed-loop system to stability (stabilising region). It is also important to note that the gain set (0, 0) is inside this region as expected, since the open-loop system is stable.

4.2 Performance criteria

Once the region of stable roots is determined, one can choose a set of gains inside this region, which satisfies a desired criterion of performance. If the idea is to use only the frequency response of the system in the controller’s design,
the following closed-loop transfer functions are considered

\[
H(j\omega) = \frac{P(j\omega)}{1 + (G_P + j\omega G_D)P(j\omega)} \quad (16)
\]

\[
S(j\omega) = \frac{1}{1 + (G_P + j\omega G_D)P(j\omega)} \quad (17)
\]

\[
T(j\omega) = \frac{(G_T + j\omega G_D)P(j\omega)}{1 + (G_P + j\omega G_D)P(j\omega)} \quad (18)
\]

where the performance criterion (16) represents the closed-loop transfer function of the system, the criterion (17) represents the sensitivity function of the system, and the criterion (18) represents the complementary sensitivity function of the system [15].

Varying the values of \(G_P\) and \(G_D\), one can calculate the expected response of the closed-loop system for each set of gains of the controller. To choose the appropriated gains, one can rely on the performance specifications below

\[
M_R = \max_{\omega \in [\omega_1, \omega_2]} |H(j\omega)| < \gamma_R \quad (19)
\]

\[
M_S = \max_{\omega \in [\omega_1, \omega_2]} |S(j\omega)| < \gamma_S \quad (20)
\]

\[
M_T = \max_{\omega \in [\omega_1, \omega_2]} |T(j\omega)| < \gamma_T \quad (21)
\]

According to [15], typical values of \(M_S\) are in the range of 1.2–2.0, and the minimum of this value \((1/M_S)\) represents a good evaluation of the controller robustness in face of
uncertainties (smaller the $M_0$, closer the distance between the Nyquist curve and the critical point $-1 + j0$). Typical values of $M_F$ are in the range of 1.0–1.5, and this value is closely related to the peak overshoot at the plant output. The values of $M_0$, as it was defined, depend on the system in study, and can vary from case to case.

The set of gains to be analysed in closed-loop are $-0.3 \leq G_P \leq 0.1 \text{ V/s/mm}$ and $-10 \leq G_D \leq 20 \text{ V/mm}$. These ranges of gain values lie within the region of stabilising gains obtained for the case of rotating shaft (590 and 1770 rpm; Fig. 6). By varying the values of the gains within these ranges, and calculating the responses of the closed-loop system, the results shown in Fig. 7 were obtained. As one can see for both rotating speeds (Figs. 7a and b), the values of the maximum amplitude of closed-loop response ($M_R$) decreases for negative derivative gains and for positive proportional gains. The values of $M_R$ also decrease in a similar way (negative derivative gains and positive proportional gains), but there are optimum regions. The values of $M_F$ always increase for larger values of derivative and proportional gains, with minimum at the origin ($G_P = G_D = 0$).

For the rotating speed of 590 rpm, one adopted the gains $G_P = 2 \text{ V/mm}$ and $G_D = -0.1 \text{ V/s/mm}$, which resulted in $\gamma_R = 0.023$, $\gamma_P = 1.006$ and $\gamma_D = 0.308$. For the rotating speed of 1770 rpm, one adopted the gains $G_P = 1 \text{ V/mm}$ and $G_D = -0.07 \text{ V/s/mm}$, which resulted in $\gamma_R = 0.031$, $\gamma_P = 1.008$ and $\gamma_D = 0.279$.

By implementing the control system experimentally, one obtains the results shown in Fig. 8. As one can see, the value of the resonance peak amplitude in closed-loop agrees with the expected values. The error found between the expected and the measured values was 4% for the rotating speed of 590 rpm, and 6% for the rotating speed of 1770 rpm. The reduction of resonance peak amplitude was 28% for the rotating speed of 590 rpm and 25% for the rotating speed of 1770 rpm.

The results shown in Fig. 8 represent the maximum reduction possible for the resonance peak amplitude of the system in study. Owing to limitations in the actuation capacity of the electromagnetic actuators, the feedback signal was easily saturated in $\pm 10 \text{ V}$, which is the maximum range of the D/A ports in the acquisition system. However, if stronger actuators are used, further reduction of resonance peak amplitude can be achieved by adopting smaller values of the derivative gains, as observed in Fig. 7.

The feedback control system was implemented in the horizontal direction. By analysing the system response in the vertical direction, that is, perpendicularly to the actuation direction (Fig. 9), one observes that the controller did not affect the results negatively. As a matter of fact, there was a reduction in the vibration levels in the vertical direction when the controller was in use.

5 Conclusion

In this work, one applied a PD-controller synthesis methodology, based on the D-decomposition technique, on a flexible shaft rotating system with electromagnetic actuators. The experimental results show that it is possible to find all the proportional and derivative gains that stabilise the system in closed-loop, based solely on the FRFs of the global system. In this case, the global system is comprised of the actuator drive, the actuators, the rotating system and the proximity sensors. No mathematical model is required and precision of results will depend only on the quality of FRF measurements. According to the resonance peak amplitude criterion, the closed-loop resonance peak response remained within a 6% margin error, for both subcritical and supercritical rotating speeds.

The methodology adopted in this work involved excitation and actuation in one single direction and the results in the perpendicular direction of actuation do not show degradation of vibration levels. Hence, for systems with low gyroscopic effects, the methodology can be applied independently in both perpendicular directions. For systems with significant gyroscopic effects, one cannot consider independent PD controllers in the perpendicular directions. Instead, one must work with PD gain matrices (with cross-coupling terms), whose methodology for finding the stabilising gains must be further investigated. Disturbance and/or sensor noise rejection is another topic that is worthy of further investigation.

Despite these drawbacks in the application of such procedure in rotating systems, the advancement of the method for finding the stabilising regions of the controller gains can be guaranteed, as provided by the rich literature on the subject.

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7 References

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